Slovak University of Technology in Bratislava
Faculty of Mechanical Engineering
17^{th} Conference on Applied Mathematics **APLIMAT 2018**

Proceedings

MEASUREMENT SYSTEM STUDIES USING ANOVA

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Abstract. Analysis of variance (ANOVA) is a standard statistical technique and can be used to analyze the measurement error and other sources of variability of data in a measurement systems study. In the analysis of variance, the variance can be decomposed into four categories: appraisers, parts, interaction between appraisers and parts and replication error due to the gage. In the paper it is studied measurement system with respect to repeatability and reproducibility. In contrast to the method of average and range, the ANOVA method allows to determine the variability of the interaction between the appraisers and the parts. The paper deals with the ANOVA method of computing repeatability and reproducibility.

Keywords: measurement system, repeatability, reproducibility, interaction, part to part variability, total variability, degrees of freedom, mean squares

Mathematics Subject Classification: C1, C12, C6.

1 Introduction

The analysis of variance method (ANOVA) is the most accurate method for quantifying repeatability and reproducibility (see [8]). In addition, the ANOVA method allows the variability of the interaction between the appraisers and the parts to be determined.

The advantages of ANOVA techniques as compared with Average and Range methods are:

- They are capable of handling any experimental set-up
- Can estimate the variances more accurately
- Extract more information (such as interaction between appraisers and parts effect) from the experimental data.

The disadvantages are that the numerical computations are more complex and users require a certain degree of statistical knowledge to interpret the results. The ANOVA method as described in the following sections is advised, especially if a computer is available.

2 ANOVA Method

The ANOVA method for measurement assurance is the same statistical technique used to analyze the affects of different factors in designed experiments. The ANOVA design used is a two-way, fixed effects model with replications. The ANOVA table is shown in Table 1.

$$SS_A = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{y}_{i..} - \bar{y}_{...})^2 = \sum_{i=1}^{a}\frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{N} \quad (1)$$

$$SS_B = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{y}_{.j.} - \bar{y}_{...})^2 = \sum_{i=1}^{a}\frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{N} \quad (2)$$

$$SS_{AB} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{y}_{ij.} - \bar{y}_{...})^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij}^2}{n} - \frac{y_{...}^2}{N} - SS_A - SS_B \quad (3)$$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(\bar{y}_{ijk} - \bar{y}_{...})^2 = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}y_{ijk}^2 - \frac{y_{...}^2}{N} \quad (4)$$

$$SS_\varepsilon = SS_T - (SS_A + SS_B + SS_{AB}) \quad (5)$$

where

a = number of appraisers,
b = number parts,
n = the number of trials and
N = total number of readings (abn).

Source of variation	Sum of Squares	Degrees of Freedom	Mean squares	F Statistic
Appraiser	SS_A	$a-1$	$MS_A = \frac{SS_A}{a-1}$	$F = \frac{SS_A}{MS_e}$
Parts	SS_B	$b-1$	$MS_B = \frac{SS_B}{b-1}$	$F = \frac{MS_B}{MS_e}$
Interaction (Appraisers, Parts)	SS_{AB}	$(a-1)(b-1)$	$MS_{AB} = \frac{SS_{AB}}{(a-1)(b-1)}$	$F = \frac{MS_{AB}}{MS_e}$
Gage Error	SS_ε	$ab(n-1)$	$MS_\varepsilon = \frac{SS_\varepsilon}{ab(n-1)}$	
Total	SS_T	$N-1$		

Tab. 1. Two-Way ANOVA Table.

When conducting a study, the recommended procedure is to use 10 parts, 3 appraisers and 2 trials, for a total of 60 measurements. Then the relevant quantities are calculated as follows:

The measurement system repeatability is

$$EV = 5{,}15\sqrt{MS_{\varepsilon}} \tag{5}$$

The measurement system reproducibility is

$$AV = 5{,}15\sqrt{\frac{MS_A - MS_{AB}}{bn}} \tag{6}$$

The interaction between the appraisers and the parts is

$$INT = 5{,}15\sqrt{\frac{MS_{AB} - MS_{\varepsilon}}{n}} \tag{7}$$

The measurement system repeatability and reproducibility is

$$R\&R = \sqrt{(EV)^2 + (AV)^2 + INT^2} \tag{8}$$

The measurement system part variability is

$$PV = 5{,}15\sqrt{\frac{MS_B - MS_{AB}}{an}} \tag{9}$$

The total measurement system variability is

$$TV = \sqrt{(R\&R)^2 + (PV)^2} \tag{10}$$

The percent of total variability accounted for by each factor is calculated as follows:

$$\%EV = 100\left[\frac{EV}{TV}\right] \tag{11}$$

$$\%AV = 100\left[\frac{AV}{TV}\right] \tag{12}$$

$$\%R\&R = 100\left[\frac{R\&R}{TV}\right] \tag{13}$$

$$\%PV = 100\left[\frac{PV}{TV}\right] \tag{14}$$

What is considered acceptable for $\%R\&R$, gives the following guidelines:

10% or less	excellent
11% to 20%	adequate
21% to 30%	marginally acceptable
over 30%	unacceptable

3 Example

The thickness [mm] of 10 parts have been measured by 3 operators, using the same measurement equipment. Each operator measured each part twice and the data is given in Table 2.

To compute the characteristics of this measurement system, the two-way ANOVA table must be completed. The sum of the 20 readings (10 parts multiplied by 2 trials) for appraiser A is 1510,2. The sum of the 20 readings for appraiser B is 1457,7. The sum of the 20 readings for appraiser C is 1598,0. The sum of all 60 readings is 4565,9. The sum of squares for the appraisers is

	Operator					
	A		B		C	
Part	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2
1	55,2	50,1	52,9	46,3	61,6	50,6
2	75,8	76,3	75,7	70,5	82,0	77,4
3	90,2	84,8	90,1	84,5	97,3	94,4
4	75,0	85,1	74,8	80,3	82,3	84,6
5	44,7	55,8	41,7	50,0	48,9	57,2
6	88,7	80,2	82,7	77,2	88,9	83,5
7	84,5	84,5	81,0	83,4	85,4	93,3
8	77,2	72,4	73,9	68,8	83,0	75,8
9	72,4	72,2	70,7	70,3	77,9	78,1
10	90,2	94,9	89,7	93,2	94,3	101,5

Tab. 2. ANOVA method example data.

$$SS_A = \frac{1810,2^2}{10\times 2} + \frac{1457,7^2}{10\times 2} + \frac{1598,0^2}{10\times 2} - \frac{4565,9^2}{60} = 502,486$$

The sum of the 6 readings for each part (3 appraisers multiplied by 2 trials) is given in Table 3 along with the square of this sum and the square of this sum divided by 6.

The sum of squares for the parts is

$$SS_B = 359\,002,9 - \frac{4565,9^2}{60} = 11\,545,52$$

Part	Sum	Sum Squared	Sum Squared/6
1	316,7	100298,9	16716,48
2	457,7	209489,3	34914,88
3	541,3	293005,7	48834,28
4	482,1	232420,4	38736,74
5	298,3	88982,89	14830,48
6	501,2	251201,4	41866,91
7	512,1	262246,4	43707,74
8	451,1	203491,2	33915,2
9	441,6	195010,6	32501,76
10	563,8	317870,4	52978,41
Total			359 002,9

Tab. 3. Part sum of squares computations.

The sum of the 2 trials for each combination of appraiser and part is given in Tab. 4 along with the square of this sum and the square of this sum divided by 2.

Part	Appraiser	Sum	Sum Squared	Sum Squared/2
1	A	105,3	11 088,1	5 544,0
2	A	152,1	23 134,4	11 567,2
3	A	175,0	30 625,0	15 312,5
4	A	160,1	25 632,0	12 816,0
5	A	100,5	10 100,3	5 050,1
6	A	168,9	28 527,2	14 263,6
7	A	169,0	28 561,0	14 280,5
8	A	149,6	22 380,2	11 190,1
9	A	144,6	20 909,2	10 454,6
10	A	185,1	34 262,0	17 131,0
1	B	99,2	9 840,6	4 920,3
2	B	146,2	21 374,4	10 687,2
3	B	174,6	30 485,2	15 242,6
4	B	155,1	24 056,0	12 028,0
5	B	91,7	8 408,9	4 204,4
6	B	159,9	25 568,0	12 784,0
7	B	164,4	27 027,4	13 513,7
8	B	142,7	20 363,3	10 181,6
9	B	141,0	19 881,0	9 940,5
10	B	182,9	33 452,4	16 726,2
1	C	112,2	12 588,8	6 294,4
2	C	159,4	25 408,4	12 704,2
3	C	191,7	36 748,9	18 374,4
4	C	166,9	27 855,6	13 927,8
5	C	106,1	11 257,2	5 628,6
6	C	172,4	29 721,8	14 860,9
7	C	178,7	31 933,7	15 966,8

8	C	158,8	25 217,4	12 608,7
9	C	156,0	24 336,0	12 168,0
10	C	195,8	38 337,6	19 168,8
Total				359 541

Tab. 4. Interaction sum of square computations.

The sum of squares for the interaction between the appraisers and the parts is

$$SS_{AB} = 359\,541 - \frac{4565,9^2}{60} - 502,5 - 11\,545,52 = 35,6$$

Squaring all 60 individual reading and summing the values gives 457 405,8. The total sum of squares is

$$SS_T = 359\,087,8 - \frac{4565,9^2}{60} = 12\,630,4$$

The sum of squares for the gage or error is

$$SS_\varepsilon = 12\,630,4 - 502,5 - 11\,545,5 - 35,6 = 546,8$$

There are 2 degrees of freedom for the appraisers, the number of appraisers minus one; 9 degrees of freedom for the parts, the number of parts minus one, 18 degrees of freedom for the interaction between the appraisers and the parts, the number of appraisers minus one multiplied by the number of parts minus one; 59 total degrees of freedom; the total number of readings minus one, and 30 degrees of freedom for the gage, total degrees of freedom minus the degrees of freedom for the appraisers minus the degrees of freedom for the parts minus the degrees of freedom for the interaction. Since the mean square error is the sum-of-square divided by degrees-of-freedom, the ANOVA table can be completed as shown in Table 5.

Source of variation	Sum of Squares	Degrees of Freedom	Mean squares	*F*-statistic	Significance
Appraiser	502,5	2	251,3	13,8	0,000057
Parts	11 545,5	9	1 282,8	70,5	0,000000
Interaction (Appraisers by Parts)	35,6	18	1,98	0,11	0,999996
Gage (Error)	546,8	30	18,2		
Total	12 630,4	59	214,1		

Tab. 5. Example ANOVA table.

The ANOVA table divides the total variability into several parts. The first part represents differences between the appraisers. The second part represents differences between the parts. The third part represents interactions between appraiser and parts. Since the P-value in the ANOVA table is greater than or equal to 0,05, there is not a statistically significant interaction between appraisers and parts at the 95,0% confidence level. The last part represents the residual error, which corresponds to Repeatability.

The significance listed in Table 5 represents the probability of Type I error. Stated another way, if the statement is made "the appraisers are a significant source of measurement variability", the probability of this statement being incorrect is 0,000057. This significance is the area under the F probability density function to the right of the computed *F*-statistic. This value can be found using the function FDIST(F-statistic, d_1, d_2) in Microsoft Excel, where d_1 and d_2 are the appropriate degrees of freedom.

Continuing with the example the relevant value are the following:

Repeatability is

$$EV = 5{,}15\sqrt{18{,}2} = 21{,}97$$

Reproducibility is

$$AV = 5{,}15\sqrt{\frac{251{,}3 - 1{,}98}{10 \times 2}} = 18{,}18$$

The interaction between the appraisers and the parts is

$$INT = 5{,}15\sqrt{\frac{1{,}98 - 18{,}2}{2}} = \text{cannot be computed}$$

Obviously the variability due to the interaction cannot be imaginary (the square root of a negative number is an imaginary number); what happened? Each mean square is an estimate subject to sampling error. In some cases the estimated variance will be negative or imaginary. In these cases, the estimated variance is zero.

The repeatability and reproducibility is

$$R\,\&\,R = \sqrt{21{,}97^2 + 18{,}18^2 + 0^2} = 28{,}517$$

The part variation is

$$PV = 5{,}15\sqrt{\frac{1\,282{,}8 - 1{,}98}{3 \times 2}} = 75{,}245$$

We now calculate confidence intervals. The results are shown in the Table 6.

	Lower Limit	5,15×Std. dev.	Upper Limit
Repeatability; *EV*	17,5701	21,987	29,3895
Reproducibility; *AV*	8,51443	18,1812	114,184
Interaction	0,0	0,0	0,0
R&R	18,8587	28,5304	115,867
parts	43,9656	75,2456	144,767

Tab. 6. 95% Confidence Intervals.

This table shows intervals equal to 5,15 times the standard deviations due to Repeatability, Reproducibility, combined R&R and variability between parts. These intervals can be expected to contain 98,9976% percent of the errors attributed to each source. For example, we would expect the measurements to deviate from the true values by +/-14,2652 due to combined R&R, an interval 28,5304 units wide. Since the estimates of variability are subject to sampling error, the confidence intervals show how precise these estimates are.

The total measurement system variability is

$$TV = \sqrt{28,517^2 + 75,245^2} = 80,468$$

Then the percent total variability is calculated as follows:

$$\%EV = 100 \times \sqrt{\frac{21,97}{80,468}} = 27,3$$

$$\%AV = 100 \times \sqrt{\frac{18,18}{80,468}} = 22,6$$

$$\%R\&R = 100 \times \sqrt{\frac{28,517}{80,468}} = 35,4$$

$$\%PV = 100 \times \sqrt{\frac{75,245}{80,468}} = 93,5$$

Since $\%R\&R$ is greater than 30, namely 35,4, we consider the gauge to be unacceptable.

4 Conclusion

ANOVA method will provide information concerning the causes of measurement system or gage variation. The repeatability is large compared to reproducibility it follows that the reasons may be:

- The instrument needs maintenance.
- The gage may need to be redesigned to be more rigid.

- The clamping or location for gaging needs to be improved.
- There is excessive within-part variation.

A fixture of some sort may be needed to help the appraiser use the gage more consistently. Since $\%R\&R$ is greater than 30, namely 35,4, we consider the gauge to be unacceptable.

Acknowledgement

The paper was supported by grant from Grant Agency of VEGA no. 1/0544/16.

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