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GAGE STUDIES FOR VARIABLES AVERAGE AND RANGE METHOD

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Abstract. There are several methods that can be used to measure gauge variability. The average and range method is widely used in industry because its calculations can be done by hand. The Average and Range method is an approach which will provide an estimate of both repeatability and reproducibility for a measurement system. This approach will allow the measurement system´s variation to be decomposed into two separate components, repeatability and reproducibility. However, variation due to the interaction between the appraiser and the part or gage is not accounted for in the analysis.

Keywords: gage, measurement system, repeatability, reproducibility, average, part to part variability, total variability

Mathematics Subject Classification: C1, C12, C6.

1 Introduction

There are several methods that can be used to measure gauge variability. In this study we will use the Average and Range method ($\bar{X}$ & R). This method is a statistical method that provides an estimate of the repeatability and reproducibility of the measurement system. Now we explain what is repeatability and reproducibility.

1.1 Repeatability

Repeatability is the variability of the measurements obtained by one person while measuring the same item repeatedly. This is also known as the inherent precision of the measurement equipment. Consider the probability density functions in Figure 1. The density functions were constructed from measurements of the thickness of a piece of metal with Gage A Gage B. The density functions demonstrate that Gage B is more repeatable than Gage A.

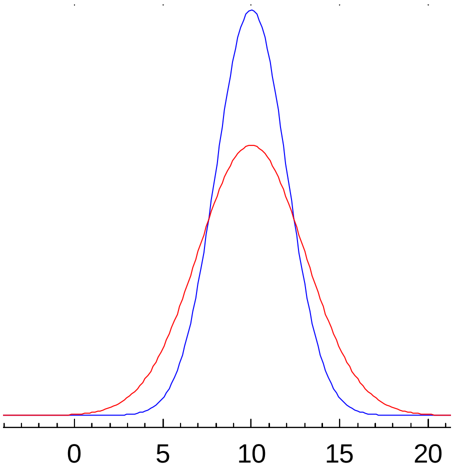

Fig. 1. Probability density functions for the thickness of 2 gages.

1.2 Reproducibility

Reproducibility is the variability of the measurement system caused by differences in operator behavior. Mathematically, it is the variability of the average values obtained by several operators while measuring the same item. Figure 2 displays the probability density functions of the measurements for three operators. The variability of the individual operators are the same, but because each operator has a different bias, the total variability of the measurement system is higher when three operators are used than when one operator is used. Figure 3 also displays the probability density functions of the measurements for three operators using the same scale as Figure 2. Notice that there is more difference in the means of the measurements shown in Figure 3 than those shown in Figure 2. The reproducibility of the system shown in Figure 3 is higher than the reproducibility of the system shown in Figure 2.

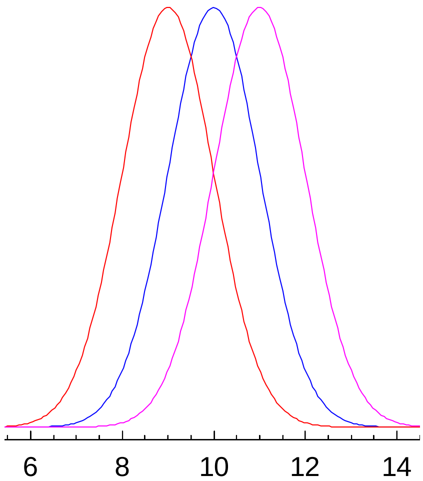

Fig. 2. Reproducibility demonstration.

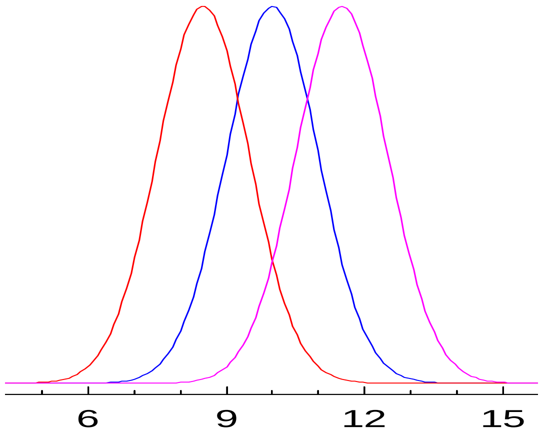

Fig. 3. Reproducibility demonstration.

The most commonly used method for computing repeatability and reproducibility is the Average and Range method.

2 Average and Range Method

All calculations described here are based upon a specified multiple of σ, where the multiple ν can be 4; 5,15; or 6. The relevant quantities are calculated as follows:

The measure of repeatability (or equipment variation), denoted by *EV*, is calculated as

$$EV = \overline{R} \times K_1 \tag{1}$$

where $\overline{R}$ is the average range and $K_1 = \nu / d_2$ is the adjustment factor.

The quantity d_2 (Duncan A. J. 1986) depends on the number of trials used to calculate a single range. In the GAGE application, the number of trials can vary from 2 to 4. Use of d_2 is valid when "operators × parts ≥ 16"; otherwise, the GAGE application uses d_2^* (Duncan 1986), which is based on the number of ranges calculated from "operators × parts" and on the number of trials.

The measure of reproducibility (or appraiser variation), denoted by *AV*, is calculated as

$$AV = \sqrt{(\overline{X}_{\text{diff}} \times K_2)^2 - \frac{(EV)^2}{nr}} \tag{2}$$

where $\overline{X}_{\text{diff}}$ is the difference between the maximum operator average and the minimum operator average, $K_2 = \nu / d_2^*$ is the adjustment factor, n is the number of parts, and r is the number of trials. Reproducibility is contaminated by gage error and is adjusted by subtracting $(EV)^2 / nr$.

The quantity d_2^* (Duncan 1986) depends on the number of operators used to calculate a single range. In the GAGE application, the number of operators can vary from 1 to 4. When there is only one operator, reproducibility is set to zero.

The measure of repeatability and reproducibility, denoted by *R&R*, is calculated as

$$R\,\&\,R = \sqrt{(EV)^2 + (AV)^2} \tag{3}$$

Part-to-part variability, denoted by PV, is calculated as

$$PV = R_p \times K_3 \tag{4}$$

where R_p is the range of part averages and $K_3 = \nu / d_2^*$ is the adjustment factor.

Here the quantity d_2^* (Duncan 1986) depends on the number of parts used to calculate a single range. In the GAGE application, the number of parts can vary from 2 to 15.

Total variability, denoted by *TV*, is based on gage R&R and part-to-part variability.

$$TV = \sqrt{(R\,\&\,R)^2 + (PV)^2} \tag{5}$$

The percent of total variability accounted for by each factor is calculated as follows:

$$\%EV = 100\left[\frac{EV}{TV}\right] \tag{6}$$

$$\%AV = 100\left[\frac{AV}{TV}\right] \tag{7}$$

$$\%R\,\&\,R = 100\left[\frac{R\,\&\,R}{TV}\right] \tag{8}$$

$$\%PV = 100\left[\frac{PV}{TV}\right] \tag{9}$$

Note that the sum of these percentages does not equal 100%. You can use these percentages to determine whether the measurement system is acceptable for its intended application.

Instead of percent of process variation, your analysis may be based on percent of tolerance. For this you must specify a tolerance value. Then $\%EV$, $\%AV$, $\%R\,\&\,R$ and $\%PV$ are calculated by substituting the tolerance value for TV (the denominator) in the preceding formulas.

What is considered acceptable for $\%R\&R$? Barrentine (1991) gives the following guidelines:

10% or less	excellent
11% to 20%	adequate
21% to 30%	marginally acceptable
over 30%	unacceptable

3 Example

The thickness, in millimeters, of 10 parts have been measured by 3 operators, using the same measurement equipment. Each operator measured each part twice, and the data is given in Table 1.

	Operator					
	A		B		C	
Part	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2
1	55,2	50,1	52,9	46,3	61,6	50,6
2	75,8	76,3	75,7	70,5	82,0	77,4
3	90,2	84,8	90,1	84,5	97,3	94,4
4	75,0	85,1	74,8	80,3	82,3	84,6
5	44,7	55,8	41,7	50,0	48,9	57,2
6	88,7	80,2	82,7	77,2	88,9	83,5
7	84,5	84,5	81,0	83,4	85,4	93,3
8	77,2	72,4	73,9	68,8	83,0	75,8
9	72,4	72,2	70,7	70,3	77,9	78,1
10	90,2	94,9	89,7	93,2	94,3	101,5

Tab. 1. Average & Range method example data.

Repeatability is computed using the average of the ranges for all appraiser and all parts. This data is given in Table 2.

	Operator								
	A			B			C		
Part	Trial 1	Trial 2	R	Trial 1	Trial 2	R	Trial 1	Trial 2	R
1	55,2	50,1	5,1	52,9	46,3	6,6	61,6	50,6	11,0
2	75,8	76,3	0,5	75,7	70,5	5,2	82,0	77,4	4,6
3	90,2	84,8	5,4	90,1	84,5	5,6	97,3	94,4	2,9
4	75,0	85,1	10,1	74,8	80,3	5,5	82,3	84,6	2,3
5	44,7	55,8	11,1	41,7	50,0	8,3	48,9	57,2	8,3
6	88,7	80,2	8,5	82,7	77,2	5,5	88,9	83,5	5,4
7	84,5	84,5	0,0	81,0	83,4	2,4	85,4	93,3	7,9
8	77,2	72,4	4,8	73,9	68,8	5,1	83,0	75,8	7,2
9	72,4	72,2	0,2	70,7	70,3	0,4	77,9	78,1	0,2
10	90,2	94,9	4,7	89,7	93,2	3,5	94,3	101,5	7,2

Tab. 2. Example problem range calculations.

The average of the 30 ranges, $\bar{R}$, is 5,18333. From Table 3, with *n* = 30 (10 parts multiplied by 3 appraisers) and *r* = 2 (2 trials), d_2 is 1,128. The constant K_1 is $5,15/1,128 = 4,5656$, then the repeatability is

$$EV = \bar{R} \times K_1 = 5,18333 \times 4,5656 = 23,665$$

d_2^*	Size of samples								
Samples	2	3	4	5	6	7	8	9	10
1	1,414	1,912	2,239	2,481	2,673	2,830	2,963	3,078	3,179
2	1,279	1,805	2,151	2,405	2,604	2,768	2,906	3,025	3,129
3	1,231	1,769	2,120	2,379	2,581	2,747	2,886	3,006	3,112
4	1,206	1,750	2,105	2,366	2,570	2,736	2,877	2,997	3,103
5	1,191	1,739	2,096	2,358	2,563	2,730	2,871	2,992	3,098
6	1,181	1,731	2,090	2,353	2,558	2,726	2,867	2,988	3,095
7	1,173	1,726	2,085	2,349	2,555	2,723	2,864	2,986	3,092
8	1,168	1,721	2,082	2,346	2,552	2,720	2,862	2,984	3,090
9	1,164	1,718	2,080	2,344	2,550	2,719	2,860	2,982	3,089
10	1,160	1,716	2,077	2,342	2,549	2,717	2,859	2,981	3,088
11	1,157	1,714	2,076	2,340	2,547	2,716	2,858	2,980	3,087
12	1,155	1,712	2,074	2,339	2,546	2,715	2,857	2,979	3,086
13	1,153	1,710	2,073	2,338	2,545	2,714	2,856	2,978	3,085
14	1,151	1,709	2,072	2,337	2,545	2,714	2,856	2,978	3,085
15	1,150	1,708	2,071	2,337	2,544	2,713	2,855	2,977	3,084
d_2									
> 15	1,128	1,693	2,059	2,326	2,534	2,704	2,847	2,970	3,078

Tab. 3. Values of d_2^* and d_2.

The average reading for appraiser A is 75,51; the average reading for appraiser B is 72,885 and the average reading for appraiser C is 79,9. To compute reproducibility, the average of

Part	Trial	Operator B	Operator C	R
1	1	52,9	61,6	8,7
2	1	75,7	82,0	6,3
3	1	90,1	97,3	7,2
4	1	74,8	82,3	7,5
5	1	41,7	48,9	7,2
6	1	82,7	88,9	6,2
7	1	81,0	85,4	4,4
8	1	73,9	83,0	9,1
9	1	70,7	77,9	7,2
10	1	89,7	94,3	4,6
1	2	46,3	50,6	4,3
2	2	70,5	77,4	6,9
3	2	84,5	94,4	9,9

4	2	80,3	84,6	4,3
5	2	50,0	57,2	7,2
6	2	77,2	83,5	6,3
7	2	83,4	93,3	9,9
8	2	68,8	75,8	7,0
9	2	70,3	78,1	7,8
10	2	93,2	101,5	8,3

Tab. 4. Reproducibility example computations.

the range between the appraiser with the smallest average reading (appraiser B in this example) and the appraiser with the largest average reading (appraiser C in this example) is needed. Table 4 shows this data.

The average of the ranges, $\bar{X}_{\text{diff}}$, is 7,015. From Table 3, with n = 1 and r = 3 for 3 appraisers, d_2^* is 1,912. The adjustment factor K_2 is equal to $5{,}15/1{,}912 = 2{,}6935$ Then the reproducibility is

$$AV = \sqrt{(7{,}015 \times 2{,}6935)^2 - \frac{(23{,}665)^2}{10 \times 2}} = 18{,}1388$$

The repeatability and reproducibility is

$$R\,\&\,R = \sqrt{(23{,}665)^2 + (18{,}1388)^2} = 29{,}8169$$

The part variability is computed using the difference between the largest and smallest part measurement, where the average is taken for all parts and appraisers. This data is shown in Table 5.

	Operator						
	A		B		C		
Part	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2	Avg
1	55,2	50,1	52,9	46,3	61,6	50,6	52,78
2	75,8	76,3	75,7	70,5	82,0	77,4	76,28
3	90,2	84,8	90,1	84,5	97,3	94,4	90,22
4	75,0	85,1	74,8	80,3	82,3	84,6	80,35
5	44,7	55,8	41,7	50,0	48,9	57,2	49,72
6	88,7	80,2	82,7	77,2	88,9	83,5	83,53
7	84,5	84,5	81,0	83,4	85,4	93,3	85,35
8	77,2	72,4	73,9	68,8	83,0	75,8	75,18
9	72,4	72,2	70,7	70,3	77,9	78,1	73,60
10	90,2	94,9	89,7	93,2	94,3	101,5	93,97

Tab. 5. Example part variability computations.

The part with the largest average belongs to part 10 and is 93,97. The lowest average belongs to part 5 and is 49,72. This difference 44,25 is the range of part averages. From Table 3, with n = 1 and r = 10 for 10 parts, d_2^* is 3,179. The adjustment factor K_3 is 1,62. Then the part variability is

$$PV = 44,25 \times 1,62 = 71,685$$

The total measurement system variability is

$$TV = \sqrt{(29,8169)^2 + (71,685)^2} = 77,6388$$

The percent total variability is calculated as follows:

$$\%EV = 100\left[\frac{23,665}{77,6388}\right] = 30,48$$

$$\%AV = 100\left[\frac{18,1388}{77,6388}\right] = 23,363$$

$$\%R\&R = 100\left[\frac{29,8169}{77,6388}\right] = 38,4$$

$$\%PV = 100\left[\frac{71,6853}{77,6388}\right] = 92,33$$

Since $\%R\&R$ is greater than 30, namely 38,4, we consider the gauge to be unacceptable.

3 Conclusion

The Average and Range method will provide information concerning the causes of measurement system or gage variation. The repeatability ($\%EV = 30,48$) is large compared to reproducibility ($\%AV = 23,363$) it follows that the reasons may be:

- The instrument needs maintenance.
- The gage may need to be redesigned to be more rigid.
- The clamping or location for gaging needs to be improved.
- There is excessive within-part variation.

A fixture of some sort may be needed to help the appraiser use the gage more consistently.
Since $\%R\&R$ is greater than 30, namely 38,4, we consider the gauge to be unacceptable.

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