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PORTFOLIO OPTIMIZATION FOR RISK AVERSE AND RISK SEEKING INVESTOR

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Abstract. The goal of this work is to optimize portfolio for two types of investors: risk averse and risk seeking investor. The main purpose of optimization is to make the highest profit as possible in investing into the mutual funds with bearable risk corresponding to investor's profile. As input to this work is selection from equity, bond and commodity funds offered by ČSOB Asset management, because of the largest offer on the Czech market.

Keywords: portfolio optimization, profit, volatility, yield, simplex method

Mathematics Subject Classification: Primary 90C05.

1 Introduction

Investor who wants to invest on capital market has three ways how to pick securities into his portfolio. The first one is by stock picking which include many analyses such as technical analysis, fundamental analysis, macroeconomic analysis and other. The goal of stock picking is to buy securities which will bring better results than a market. The second one is to buy an index as DJIA, S&P 500 etc. Investor will reduce a risk but potential yield too and in the crisis he will report a massive loses. The third one is by using some algorithm such as Simplex method. [2] [7]

Collective investment is based on the principle of association of smaller investors, which brings more efficient asset management, cost reduction, risk diversification. Another benefit is investor's access to markets, which are not reserved to small investors, either due to high transaction costs or high risk for an inexperienced investor. The history of collective investment dates back to the time of the ancient Romans, who have been grouping their businesses to limit the loss of an individual. A similar principle is also applied, for example, in insurance companies where from collected assets from the group (premium) are covered individual damages. [5]

Risk assessment and risk evaluation stems from the entity's attitudes to risk. Different subjects can evaluate the same phenomenon differently. One will evaluate the phenomenon favourably, the other unfavourably. We can divide investors by maximum risk they are willing to take on risk seeking (dynamic) and risk averse (conservative) ones.

The goal of this work is to optimize portfolio for two types of investors: risk averse and risk seeking investor. Solution of portfolio optimization problem is provided by transformation into the linear programming task. Two different sets of limitations have been set for each type of investor. The final composition of portfolio is set by maximizing of profit function.

2 Financial Background

2.1 Mutual Funds

The operation of mutual funds is based on collective investment, which is based on the principle of the association of smaller investors that results in more efficient asset management, cost reduction and risk diversification. Another investor's benefit is an access to markets which are not reserved to small investors either because of high transaction costs or high risk for an inexperienced investor.

But how does such a mutual fund work in practice? Investors invest any amounts into the fund and buy the so-called units emitted by the fund. Therefore, the total assets of the fund increase, but the number of units is proportionally increased, so the price of the individual unit does not change by the transaction. If an investor wishes to withdraw invested money from a mutual fund, this process works exactly the opposite. The fund's total assets will be reduced and the units repurchased back by the fund will cease to exist again. For this purpose, the funds must hold part of their assets (around 5 %) in the deposits. [6]

2.2 Performance and Risk Analysis

If you are selecting a fund, it is important not to blindly choose the one fund, which an "investment adviser" recommends in the bank. Choosing fund only by historical yields is not the wisest idea, as the market does not have memory (past revenues will not secure future ones). It is important to objectively analyze the performance and risk of the fund, as described below.

Volatility

By using volatility, it is possible to measure the fund's risk. This value reflects the rate at which the fund's yield fluctuates around the mean value. In the case of mutual funds, it is often possible to find historical volatility in the monthly reports (calculated from historical data), which is calculated by using the equation:

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\ln\frac{s_i}{s_{i-1}} - \mu\right)^2}, \quad (1)$$

where $\mu = \frac{1}{n}\sum_{i=1}^{n} \ln\frac{S_i}{S_{i-1}}$, S_i is the value of the underlying asset (in our case, units of mutual fund) at time i and n is the number of observations. It is therefore a standard deviation from the logarithm of the fund's returns for a certain period. This value is further annualized due to better comparative capabilities by relation$\sigma_{p.a.} = \sigma\sqrt{h}$, where h is the number of trading periods per year. [7]

Sharpe ratio

Author of the Sharpe Index (William Sharpe) wanted to compare funds with the same yield by the risk. Thus he defined the Sharpe index, which is defined as the ratio of the yield of the asset over the safe interest rate and its risk:

$$S = (R_a - RF)/\sigma_a. \tag{2}$$

Funds with this index of 1 or more are very few and they made very good results. [7] [8]

2.3 Simplex Method

The Simplex method is an iterative computational procedure which can be used for finding an optimal solution for a linear programming task. The base principle is explained in the following diagram.

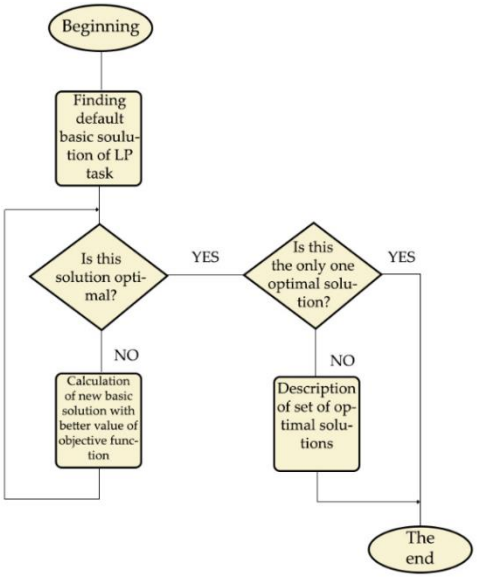

Fig. 1. Scheme of Simplex method.

In the first phase of this algorithm, is necessary to find the default base solution. Then the Simplex method in each step calculates a solution with better or at least the same value of an objective function. In case of maximizing the objective function, this value is higher, in case of minimalizing the objective function, the value is smaller. This method must lead to finding the base solution with the best value of the objective function, or to determine that there is no such solution. This solution must be optimal in compliance with the base sentence of linear programming. [3] [4]

In some cases, the first phase of finding the initial base solution is so easy that it is dropped and the algorithm continues with an iterative process that leads to the optimization of the objective z function. In this case the method is being called a one-phase simplex method. If non-trivial calculations are needed to find the initial base solution, it is a two-phase simplex method. [3]

3 Problem Formulation

Two investors are defined: conservative one and dynamic one. Both investors want to maximize their profits by investing in selected eleven mutual funds offered by ČSOB Asset Management under the following conditions:

Conservative investor

- In mutual funds, will be invested 100% of all money;
- Share of one mutual fund in portfolio can't exceed 25% of the whole portfolio;
- The weighted arithmetic mean of the standard deviations (volatility) of the individual funds in the portfolio must be less than or equal to 0.06;
- At least 50% of the portfolio must be composed of funds with a standard deviation of no more than 0.025;
- At least 20% of the portfolio must be composed of funds which have a market correlation close to zero.

Dynamic investor

- In mutual funds, will be invested 100% of all money;
- Share of one mutual fund in portfolio can't exceed 25% of the whole portfolio;
- The weighted arithmetic mean of the standard deviations (volatility) of the individual funds in the portfolio must be less than or equal to 0.12;
- At least 20% of the portfolio must be composed of funds with a standard deviation of no more than 0.05;
- At least 20% of the portfolio must be composed of funds which have a market correlation close to zero.

Index	Fund	Yield (c)	Standard deviation (σ)	Sharpe index
1	ČSOB Akciový	16,4 %	11,0 %	1,45
2	ČSOB Akciový Amerika	30,8 %	14,6 %	2,07
3	ČSOB Akciový realitní	6,1 %	14,3 %	0,39
4	KBC Equity Fund Food and Personal	5,6 %	10,6 %	0,49
5	ČSOB Komoditní	20,6 %	16,1 %	1,24
6	ČSOB Střednědobých dluhopisů	-1,0 %	0,8 %	-1,99
7	KBC Bonds High Interest	3,7 %	4,4 %	0,73
8	KBC Bonds Corporates EURO	4,5 %	2,2 %	1,87
9	KBC Multi Interest ČSOB CZK Medium	0,7 %	0,4 %	0,42
10	Index Fund Euroland	18,7 %	19,1 %	0,95
11	ČSOB Akciový BRIC	39,9 %	16,2 %	2,44

Tab. 1. Basic characteristics of selected funds.

Selected input funds are described in Table 1. The standard deviation is annualized over the preceding 12 months. For the purpose of this issue, yield of the fund is taken as the historical yield for the preceding 12 months. Fund of food, personal products and commodity fund are considered as funds with a very low correlation to the market.

4 Transformation into Linear Programming Model

First of all, it is necessary to define x_i as a share of the i-th fund in the portfolio. This variable is unknown and the goal of the task is to find all x_i that maximize profit of the portfolio and that is in compliance with investor's criteria. Since the goal is to maximize profit, the objective function will take the form of:

$$\text{maximize } \sum_{i=1}^{11} x_i \cdot c_i, \text{ for each } i \text{ applies } x_i \geq 0 \tag{3}$$

Restrictive conditions for a conservative user are:

$$\begin{gathered} \sum_{i=1}^{11} x_i = 1 \\ x_i \leq 0{,}25 \\ \sum_{i=1}^{11} x_i \cdot \sigma_i \leq 0{,}06 \\ x_6 + x_8 + x_9 \geq 0{,}5 \\ x_4 + x_5 \geq 0{,}2 \end{gathered} \tag{4}$$

For dynamic user, the restrictive conditions will be:

$$\begin{gathered} \sum_{i=1}^{11} x_i = 1 \\ x_i \leq 0{,}25 \\ \sum_{i=1}^{11} x_i \cdot \sigma_i \leq 0{,}12 \\ x_6 + x_7 + x_8 + x_9 \geq 0{,}2 \\ x_4 + x_5 \geq 0{,}2 \end{gathered} \tag{5}$$

5 Solution

The task was solved by using already mentioned Simplex method in the Solver tool of MS Excel 2016. The following table describes the portfolio of the conservative and dynamic investor.

Index	Fund	Conservative investor: share [x]	Dynamic investor: share [x]
1	ČSOB Akciový	0,0 %	4,9 %
2	ČSOB Akciový Amerika	0,0 %	25,0 %
3	ČSOB Akciový realitní	0,0 %	0,0 %
4	KBC Equity Fund Food and Personal	0,0 %	0,0 %
5	ČSOB komoditní	20,0 %	20,0 %
6	ČSOB Střednědobých dluhopisů	17,5 %	0,0 %
7	KBC Bonds High Interest	0,0 %	0,0 %
8	KBC Bonds Corporates EURO	25,0 %	25,0 %
9	KBC Multi Interest ČSOB CZK Medium	25,0 %	0,1 %
10	Index Fund Euroland	0,0 %	0,0 %
11	ČSOB Akciový BRIC	12,5 %	25,0 %

Tab. 2. Investor's portfolio.

In the case of a conservative investor, this portfolio has a standard deviation of 4.5 %, and in the period 23.2.2016 to 23.2.2017, the yield was 10.2%, resulting in a Sharpe index of 2.16. The dynamic investor portfolio has a standard deviation of 8.78%, reaching 23.7 % yield in the same period. Sharpe index is in this case 2.64. The following chart compares the earnings of a conservative and dynamic investor from February 23, 2016 to February 23, 2017.

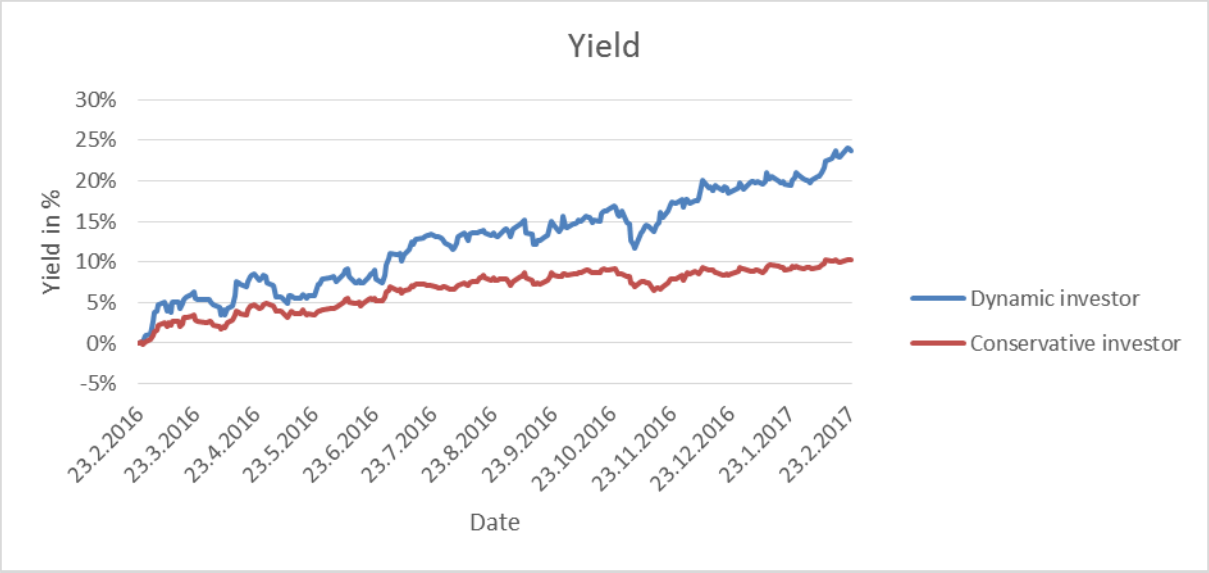

Fig. 2. Comparison of the conservative investor's and dynamic investor's portfolio.

6 Conclusion

The goal of the work was to create a portfolio of mutual funds which is in compliance with investor's criteria and which maximize profit. For the purpose of solving this problem was used the Simplex method. The data related to mutual funds has been obtained from the ČSOB company website.

The dynamic investor's portfolio achieved higher yield than conservative's portfolio, but with greater volatility. The dynamic investor portfolio shows even better Sharpe index than any of the funds in the portfolio. This is caused by diversification of nonsystematic risk.

Future improvement of this algorithm could be by adding an ability to change portfolio in real time which would depend on changes on market. This could protect investors' money in case of depression and make better profit in upturn.

Acknowledgement

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