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B3/S23 DESCENDING A STAIRCASE NO. 2

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Abstract. This artwork features gliders in cellular automatons playing Conway's game of life. Each playing field is one basic tile from the fashion pattern known as Pied de poule or Houndstooth. The pattern's basic tile has a contour consisting of straight line segments and staircases. In the context of the tessellation typology, it must be considered a hexagon. In our work of art, the basic tile is invisibly glued in a manner which is dictated by the tessellation type, thus implementing a Klein bottle topology, as an alternative to the more classical torus. The Klein bottle appears invisible but it comes to life because of the glider. The work is a tribute to the famous painting Nude Descending a Staircase No. 2, created by Marcel by Duchamp in 1912. Several other hints to Duchamp's work are embedded in the artwork. The author has a personal passion for Pied de poule, which is a rich source of aesthetic qualities in fashion and at the same time a playing field for various types mathematical recreations.

Keywords: topology, Klein bottle, Pied de poule, Marcel Duchamp, game of life

Mathematics Subject Classification: Primary: 00A66; Secondary:54E99, 05B45, 37B15.

1 Introduction

In this artwork, we propose a synthesis of Conway's game of life, topology, and art history, in particular, elements from Marcel Duchamp's work. The Pied de poule is an element added by the author and it serves to integrate the other elements of the synthesis. In this section, we describe various aspects of these ingredients and in the next section we describe the first version of our artwork, which is based on a torus topology. In the section after that, we describe B3/S23 Descending a Staircase No. 2, which is a set of Klein bottles.

The work fits in the category of "Generative art", embracing what the late Professor Mauro Francaviglia, to whom this session on Mathematics and Art is dedicated, and Marcella Giulia Lorenzi [1] wrote in 2007: (quote) *The defining trait of Generative Art is rather that the artist establishes a system, which can generate a number of possible forms rather than one single finished form. The role of the artist is to construct, initiate or merely select the frame of procedures for the generation of possible expressions; the constructions of art-making systems substitute the making of static forms while in other cases the systems rely on input* *from human actors or information feeds* (end quote). In 2017 we presented already a generative art-based work [2] in which Pied de poules came to life inside a cellular automaton. In a certain way, the present proposal works the other way around: the cellular automatons are alive inside Pied de poules.

1.1 Conway's Game of Life

A well-known form of entertainment for programmers and mathematicians is the development of various versions of Conway's game of Life and studying the behavior of Life forms such as gliders, oscillators, and still-lives. Usually, the playing field's cell structure is considered infinite or has a torus topology. Gardner described Life in 1970 [3].

Each cell has eight neighbors, which is known as a Moore neighborhood. The rule of Life is well-known (the B3/S23 rule): a new cell is Born on a blank square if and only if it is surrounded by exactly three alive cells; a cell Survives if and only if it is surrounded by two or three alive cells; if a cell is surrounded by less than two or more than three cells it dies (by loneliness or overpopulation, respectively). All kinds of patterns "live" in this artificial world, one of the simplest examples being the glider found by Richard K. Guy in 1970. In the artwork proposed (Sections 2 and 3), we use such gliders. Each glider has five alive cells and operates on a moving area of 18 cells: the live cells and their relevant neighborhood cells. See Fig. 1. The author has met John Conway in 2015 and is still fascinated by his mathematical creativity. Conway did not only invent the game of Life, but also the surreal numbers and much more [4].

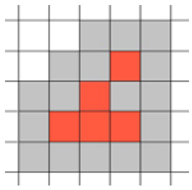

Fig. 1. Glider and its neighbourhood.

1.2 Topology

We assume the reader is somewhat familiar with general topology and knows constructions such as the torus and the Klein bottle. The torus has important applications in computer games, for example the Pacman game has the torus topology as its playing field. What topology and Conway's game of Life have in common is the notion of "neighborhood". Life has a discrete neighbourhood (on the grid). Topology is based on open sets, defining a topological space as a set of points with their neighborhoods for each point, satisfying certain axioms. As the notion of neighborhood works locally, there is considerable freedom in choosing the long-distance form-giving of a playing field. The idea of running Life on special topologies is not new (for example on www.youtube.com/watch?v=lxIeaotWIks, Life on a torus is presented). The practical advantage of the torus topology is that there is no need for special Life rules coping with "what to do at the edge?"

1.3 Pied de Poule

The essential mathematics of Pied de poule was described by Feijs in 2012 [5]. There is not a single pattern, but a family: one for each N = 1, 2, 3, etc., as shown in Fig. 2. The pattern arises naturally in weaving when a so-called twill binding is used (two over/two under, or three over/three under, etcetera).

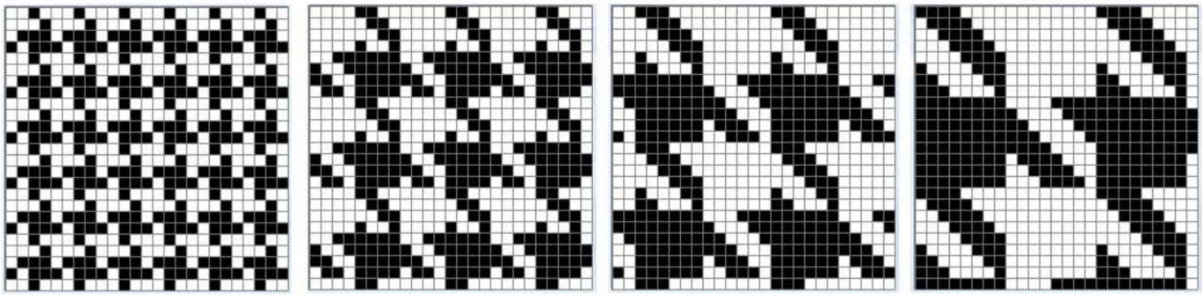

Fig. 2. Pied de poule patterns for $N = 1$, $N = 2$, $N = 3$ and $N = 4$.

In contemporary fashion, Pied de poule is still very much alive. It is an important cultural and mathematical phenomenon.

The Pied de poule patterns can be considered as tessellations, and in the 2012 Bridges paper, the typology according to Heesch and Kienzle's tessellation theory [6] is clarified. Classical Pied de poule has Heesch and Kienzle type TTTTTT, three pairs of Translated edges[1]. We show the tessellation network, omitting the black and white colour filling, in Fig. 3. For the purposes of the present paper it is important to note that the contour of each basic tile consists of six edges and six vertices. Topologically it is a hexagon.

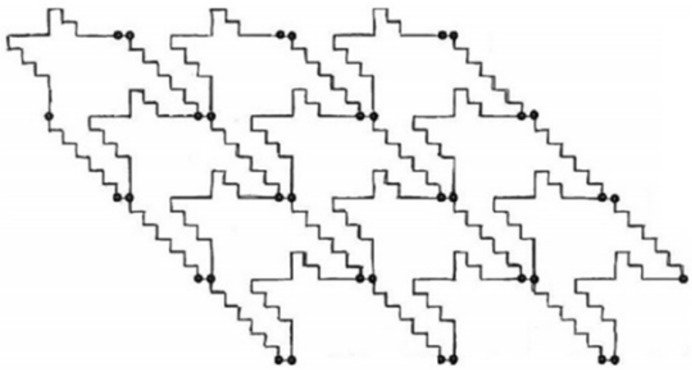

Fig. 3. Pied de poule ($N = 3$) viewed as a tessellation.

1.5 Marcel Duchamp

Marcel Duchamp (1887 - 1968) is famous for his proposition of "ready-mades", artworks which are in fact everyday objects but become a work of art, just by the artist's choice. We

[1] Heesch and Kienzle give the following recipe for this type: (quote) Verschiebe die Willkürliche Linie *AB* nach *DC*. Verschiebe die willkürliche Linie *AF* (*F* beliebig) nach *EC* (Translationsvektor *AE*). Verschiebe eine dritte willkürliche Linie *BE* nach *FD* (Translationsvektor *BF*). (end quote)

shall come back to a few of the ready-mades later. Here we like to mention the famous artwork Nude Descending a Staircase, No. 2 (French: Nu descendant un escalier n° 2) he painted in 1912. It was refused from an art show in Paris and later caused a lot of sensation in New York. Fig. 4 is a reproduction (the original is 147 cm × 89.2 cm).

Fig. 4. Nude Descending a Staircase, No. 2 (1912).

2 The artwork (initial version)

The artwork is a projected two-dimensional program output. It shows one basic Pied de poule tile, whose edges are invisibly glued in a torus topology and it shows a glider who lives in the rectilinear grid inside the tile. The dead cells are white, the alive cells are in various shades of skin color-like pink. The gluing of the edges is precisely reflecting the tessellation type of classical Pied de poule.

The tile is projected onto a manikin, as shown in Fig. 5. She wears an accessory in Pied de poule motive (a western bow tie). In this version, there is just one basic tile (but in Section 3 we propose a more sophisticated version where the manikin is “dressed” by a multi-tile set of Life universes).

Fig. 5. B3/S23 Descending a Staircase No. 1 (1917).

For the paper, the dynamic behavior is shown here as a sequence of six snapshots in Fig. 6. The wiggling behavior of the glider appears precisely like walking down a staircase. This staircase is formed by the leftmost tail of the Pied de poule tile.

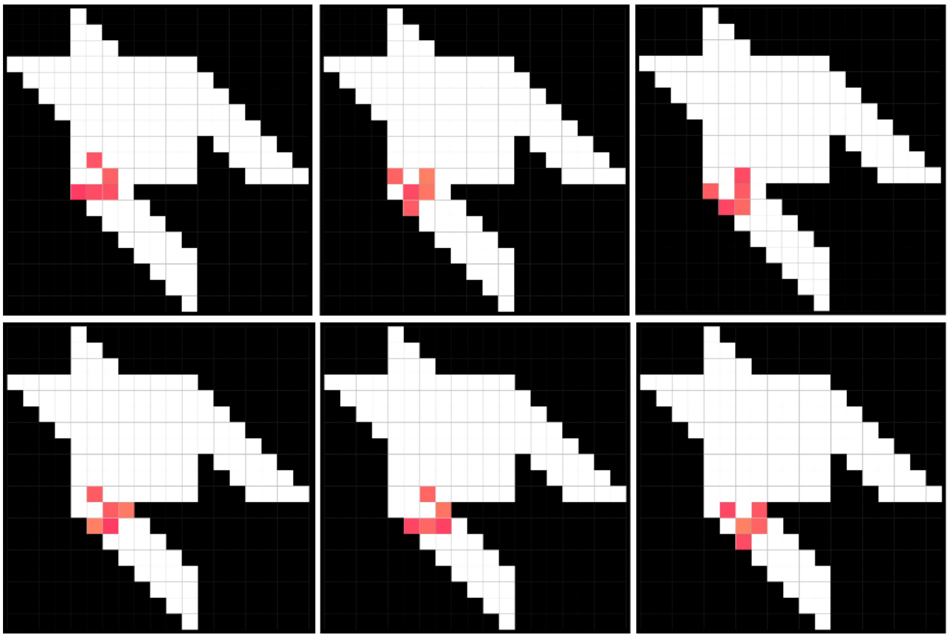

Fig. 6. Pink glider walking down the staircase.

The title of my work refers to the famous work of Marcel Duchamp, which has been re-interpreted in many forms and many media. The glider is the nude, although it is not anthropomorphic, its color suggests nudity and the glider walks down the staircase. It is a strong dynamic image. Note that pixelation is well-known technique for hiding nudity (see Fig. 7).

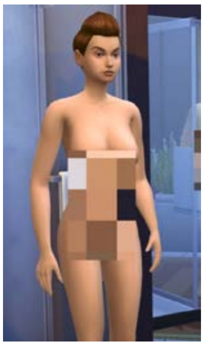

Fig.7. Pixelation hiding a Sim's nudity.

Although it appears obvious that the glider can walk the stairs, this is actually not trivial, as the neighborhood folds over the edges of the basic tile. Yet it goes well because of the glued topology which has no edges.

One would guess that the gluing of the edges is the same as gluing a rectangular playing field into a torus, as in Pacman. But we must be careful because the tessellation type tells us that the tile is not a quadrilateral but resembles a hexagon of type $abca^{-1}b^{-1}c^{-1}$. The general solution for gluing the edges of a hexagon topologically into a torus is shown in Fig. 8. The picture is adapted from www.math.cornell.edu/~mec/Winter2009/Victor/part1.htm. The technique is called an "Indian burn" in math.stackexchange.com/questions/53454.

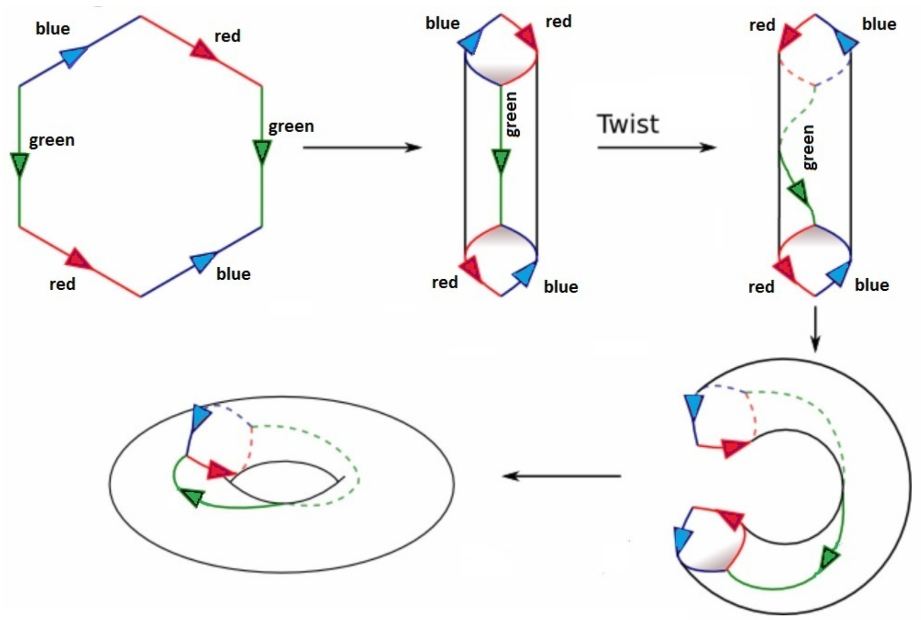

Fig. 8. Gluing the edges of a hexagon into a torus.

For the Pied de poule hexagon, only a small twist would suffice (the sides of the hexagon are not equally long). The hexagon of the pied de poule tile is abstractly shown in Fig. 8 (upper left). The edges to be glued in Fig. 8 are shown in the same color (blue to blue, red to red and green to green, no reversals). For the Life simulation, the connections are made in software, rewiring the grid of cells. In a computer program it works as follows:

```
cells[4][11].west ← cells[11][3];
cells[4][11].south ← cells[12][4];
cells[5][12].west ← cells[12][4];
cells[5][12].south ← cells[13][5];
etc.
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The following figure (Fig. 9) shows the detailed edges to be glued as indicated by the colors: blue goes to blue, red to red, green to green, without any reversals.

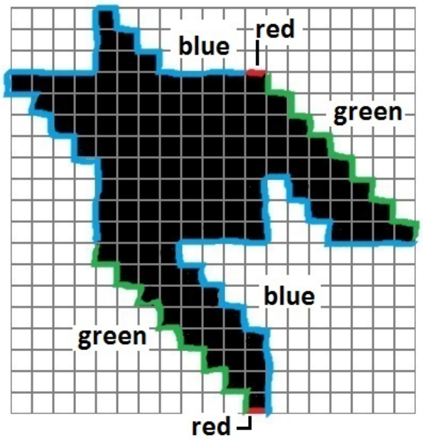

Fig. 9. Corresponding sides of the Pied the poule tile (N = 4).

3 Second version of the artwork

The torus topology is one of several options. Already in our 2012 work, we noticed that the very same basic tile could be configured in multiple ways to create new Pied de poule-like fashion patterns. In 2012 we thus found and published a new network topology. Here we propose yet another new pattern. The Heesch-Kienzle type is $TG_1G_2TG_2G_1$, which means one pair of Translated edges and two pairs of Glide reflected edges[2], see [6]. This new network topology can be folded as well, gluing edges and then it becomes a Klein bottle! The new pattern, which defines a way of gluing the edges, is shown in Fig. 10.

[2] Heesch and Kienzle give the following recipe for this type, in German: (quote) Verschiebe die Willkürliche Linie *AB* nach *DC*. Gleitspiegele die weitere willkürliche Linie *AF* (*F* beliebig) nach *EB*. Gleitspiegele eine dritte willkurliche Linie *FD* nach *CE* (Gleitspiegelachsen H_1I_1, H_2I_2 senkrecht *AD* in gleichen Abständen von *B* und *F*, bzw. von *D* und *E*) (end quote)

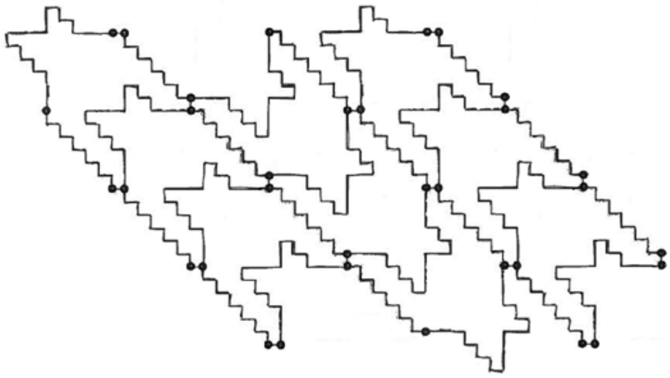

Fig. 10. Klein bottle-topology "Pied de poule" ($N = 3$).

Filling the tiles alternatingly with black and white, a new "Pied de poule" appears, as shown in Fig. 11 (for N = 4). Superficially it looks the same as the innovative pattern of 2012 [5], but it has another network type.

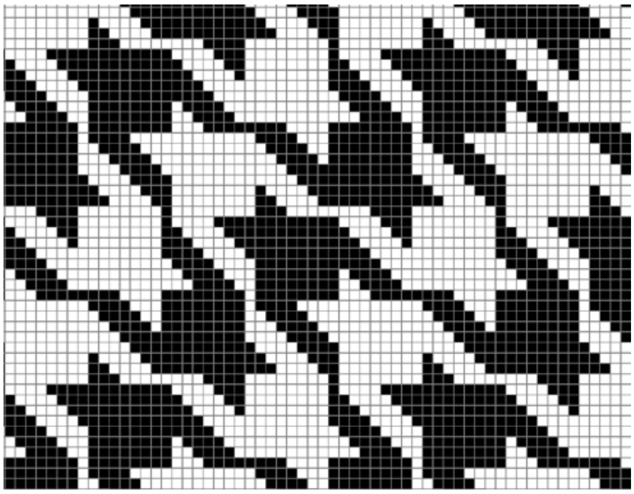

Fig. 11. New "Pied de poule" based on a Klein bottle-topology network ($N = 4$).

Some of the basic tiles are mirrored now. If we consider them as puzzle pieces, they would be upside down. But if we fold everything to a single Klein bottle, these upside-down puzzle pieces correspond to the backside of the surface (just like on a Möbius strip, there are continuous walks where one passes along the other side of the visible surface). In the program output, the active backside ("verso") cells are shown dark green (instead of pink).

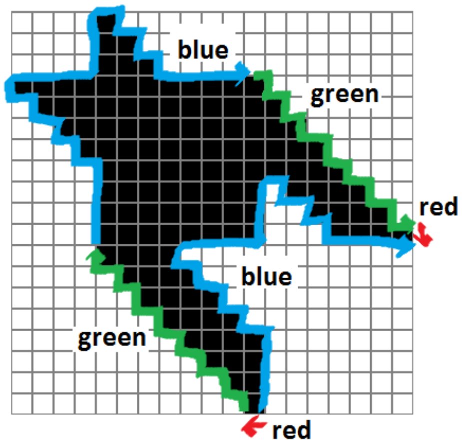

Fig. 12. Matching edges for Klein bottle-topology "Pied de poule" ($N = 4$). The blue edges have the same direction, whereas the red and green edge pairs have to be glued reversely.

It is well-known how to fold a reverse-edges square or rectangle into a Klein bottle. The construction is given in Fig. 13 (the figure is adapted from quibb.blogspot.nl/2011/04/). But here we are dealing with a hexagon again: the tessellation type dictates that the tile is essentially a hexagon of type $abca^{-1}cb$. Referring to Fig.12, *a* is blue (the asymmetric two-horned head), *b* is green (the staircase flanks) and *c* is red (the one-step end of the tails). We cannot simply take each pair of green and red edges (*b* and *c*) together to behave as one edge, as *bc* differs from *cb*. But we can apply the Indian burn method demonstrated in Section 2 (Fig. 8) once more, and then the very same Klein bottle construction applies again.

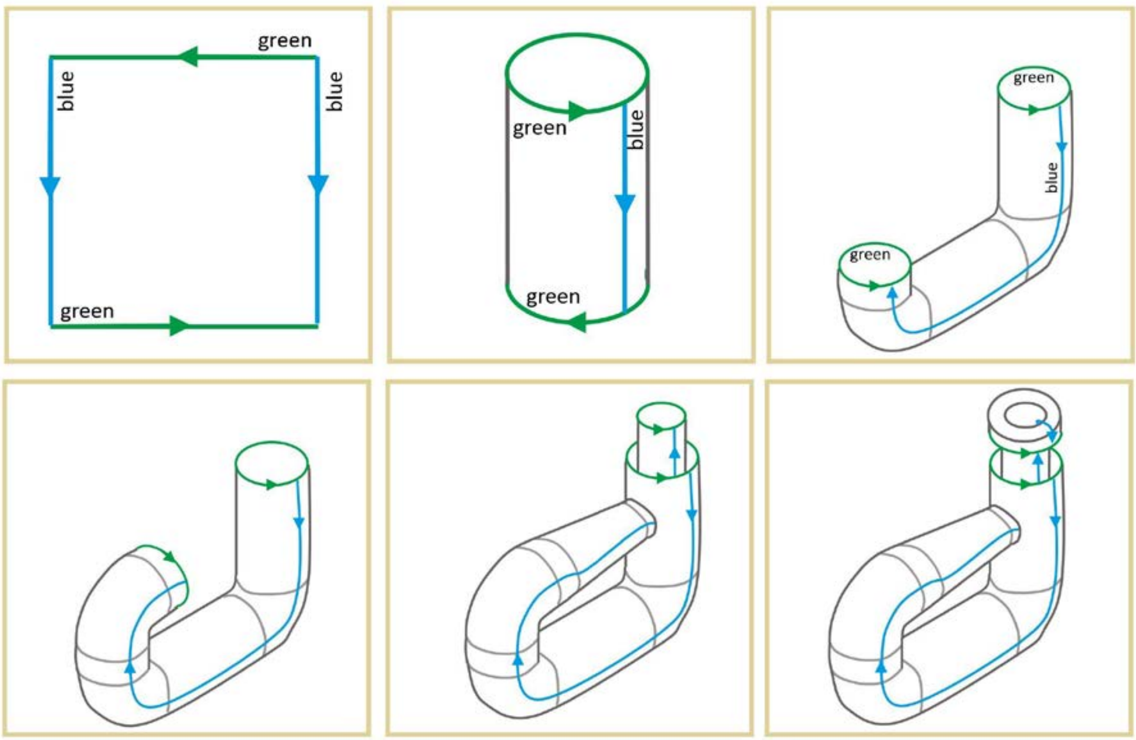

Fig. 13. Gluing a square into a Klein bottle.

Turning to the installation: instead of one basic tile, we project ten of them. The upper tile, larger than the others, shows the glider (the nude) descending the staircase. Although the topology is a Klein bottle now, not a torus, the glider walks along the staircase as before.

Then how can we "see" the Klein bottle? That is what the other basic tiles do. Each tile is a Life universe. They are not connected, but all run as an isolated Life-on-a-Klein bottle. The initial position of the gliders on the 2nd till 10th tile are different. There is also a beehive (the "eye" still-life on the verso side of I the second tile) and a "blinker" oscillator (in the third tile, verso). Instead of the South-east going nude, most of these gliders are North-east or North-west going. The latter gliders show a variety of pathways along the Klein bottle and some of them are seen to travel both the front side and the verso side of the surface. The variety of gliders on the ten Klein bottles visually confirm that the basic tiles are Klein bottles. We see the pathways of the gliders, sometimes jumping when they pass the outer zigzags and changing from pink to dark green. To make it explicit that the tiles are not connected to their neighbors, the Life execution threads run asynchronously.

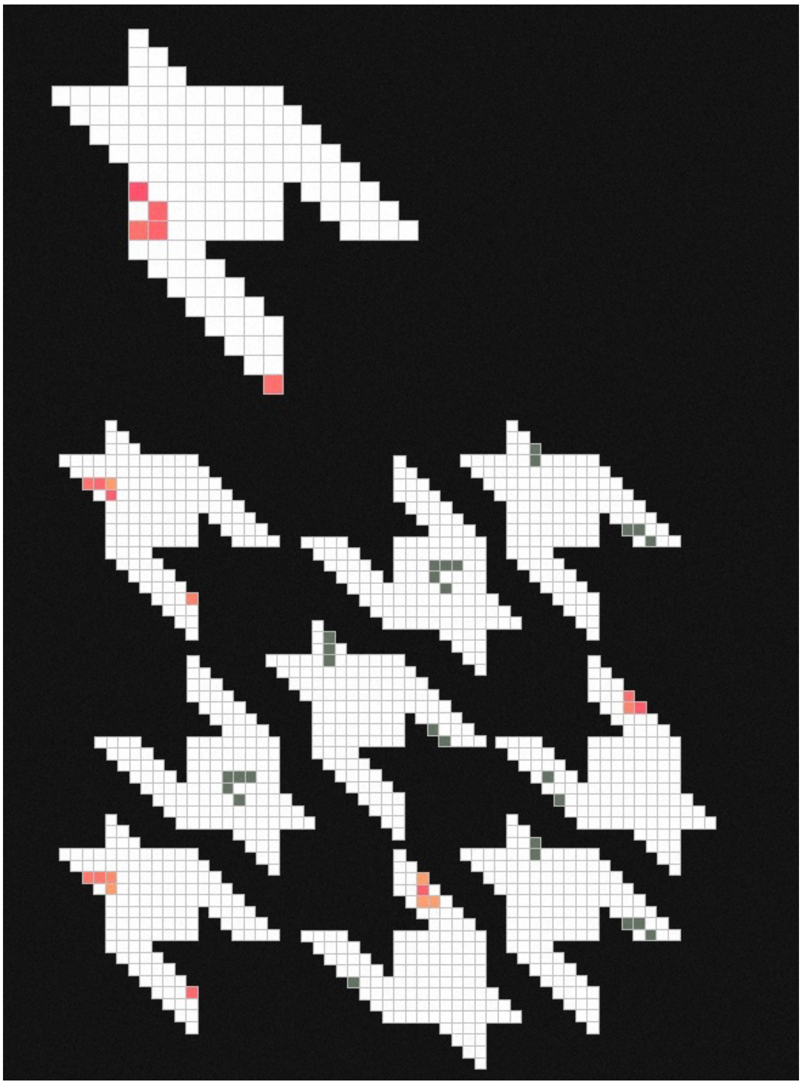

Fig. 14. Snapshot for the installation: B3/S23 Descending a Staircase No. 2 (2018). It is to be projected again, somewhat similar to No. 1.

The work has several other links to various works of Duchamp:

- The bottle rack, a ready-made produced by Duchamp in 1914. In a sense, our installation is a bottle rack, as it holds ten (Klein) bottles.
- The Fountain, in fact, a porcelain urinal, a ready-made by Duchamp in 1917. It does not take great imagination to see a connection between the Fountain and the Klein bottle. Duchamp was certainly aware of the Klein bottle. As Giunti [7] puts it: (quote) *Let's now consider the famous Fountain (1917) [ ]. It seems to me that it can be seen as a transversal section of a kleinian bottle. The neck for the connection with the water pipe (in the foreground of the historic photo) would be the equivalent of the kleinian bottle neck--plunging in its own belly (here the sectioned part) would reconnect with the holes of the draining (corresponding to the introverted bottom of the kleinian bottle)* (end quote).
- The Bride Stripped Bare by Her Bachelors, Even (La mariée mise à nu par ses célibataires, même), most often called The Large Glass (Le Grand Verre), an artwork produced by Marcel Duchamp from 1915 to 1923. In our work, there is one nude (the bride) and nine bachelors (the nine other Klein bottles, whose gliders are mostly moving upward rather than downwards). As Hopkins [8] discusses, it remains unclear whether the nude in Nude Descending a Staircase N^{o} 2 was intended male or female, but here we imagine the glider of the upper tile to be a female.

We conclude with a few implementation notes. The Life engine is tailor-made for this application using object-oriented programming in Processing (essentially Java). There are classes for cells and for Pied de poules. Each cell *c* has fields *c*.north, *c*.east, *c*.south and *c*.west. On a torus, the neighborhood of cell *c* consists of *c*.north, *c*.north.east, *c*.east, *c*.south, etc. (8 cells). On the Klein bottle, some special care is needed because once an edge is crossed, the roles of north and east are swapped and similarly for the roles of south and west. The program consists of about 750 lines of code. Efficiency is not a problem (using an HP ZBook laptop).

Acknowledgements

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References

[1] LORENZI, MARCELLA GUILIA and FRANCAVIGLIA, MAURO. Painting with light: generative artworks or “setting in motion” the fourth dimension. In *Proceedings of the 10th Generative Art Conference*, GA2007, Milano (pp. 12-14).

[2] FEIJS, LOE and TOETERS, MARINA. A Cellular Automaton for Pied-de-poule (Houndstooth). *Proceedings of Bridges 2017: Mathematics, Art, Music, Architecture, Education, Culture* (pp 403--406). Tessellations Publishing, Phoenix, Arizona.

[3] GARDNER, MARTIN (October 1970). "Mathematical Games – The fantastic combinations of John Conway's new solitaire game "life"". *Scientific American*. 223: 120–123.

[4] ROBERTS, SIOBHAN. Genius At Play: *The Curious Mind of John Horton Conway*. Bloomsbury USA (2015).

[5] FEIJS, LOE (2012). Geometry and computation of Houndstooth (Pied-de-poule). In Robert Bosch, Reza Sarhangi & Douglas McKenna (Eds.), *Proceedings of the Bridges Towson Conference 2012: Mathematics, Music, Art, Architecture, Culture*.

[6] HEESCH, HEINRICH and KIENZLE, OTTO (1963). *Flächenschluss; System der Formen lückenlos aneinanderschliessender Flachteile*. Springer Verlag, Berlin.

[7] GIUNTI, ROBERTO. *R.rO.S.E.Sel.A.Vy* by Roberto Giunti, Section 4b. Bottle in Art in Bottle (p. 13), www.toutfait.com/issues/volume2/issue_4/articles/giunti/giunti13.html

[8] HOPKINS, DAVID. *Marcel Duchamp and Max Ernst: The Bride Shared*. Clarendon Press (1998).

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