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MODELLING OF MATRIX CONVERTER SYSTEM WITH FICTITIOUS INTERLINK FUNCTION USING INSTANTANEOUS STATE CALCULATION

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Abstract. The paper deals with the analysis of matrix converter system using discrete instantaneous state calculation method. Because of impulse character of matrix converter, the space-state equations are transformed and discretized into discrete form. A virtual model of the matrix converter system uses fictitious interlink, and due to non-linear loads, the exciting fictitious functions are used for the creation of the discrete model. A new modified method of solution makes possible to determine the state of the system instantaneously. Results of theoretical analysis are confirmed by numerical computer simulation.

Keywords: matrix converter, modeling and simulation, circuit analysis, state-space equation, linear discrete control

Mathematics Subject Classification: Primary 15A05; Secondary 65R10.

1 Introduction - basic description of matrix converter system

Matrix converters are an important part of the field of conversion and conditioning of electrical energy [1]. One of them is matrix converter system (MCS) which converts the three-phase system to five-phase one ([3x5]). Such a system can be considered as a direct or indirect system [2-3]. The basic topology of [3x5] matrix converter system is depicted in fig. 1.

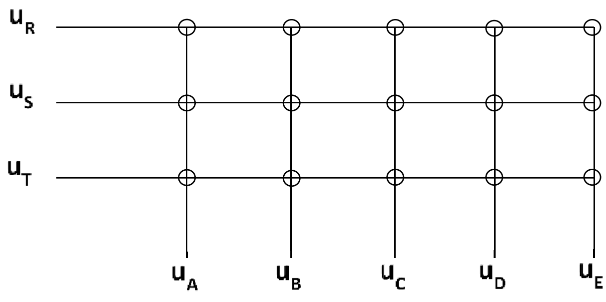

Fig. 1. Matrix topology of [3x5] matrix converter system (every single node comprises a bidirectional electronic switch).

Corresponding equivalent electrical scheme of [3x5] MCS is depicted in fig. 2.

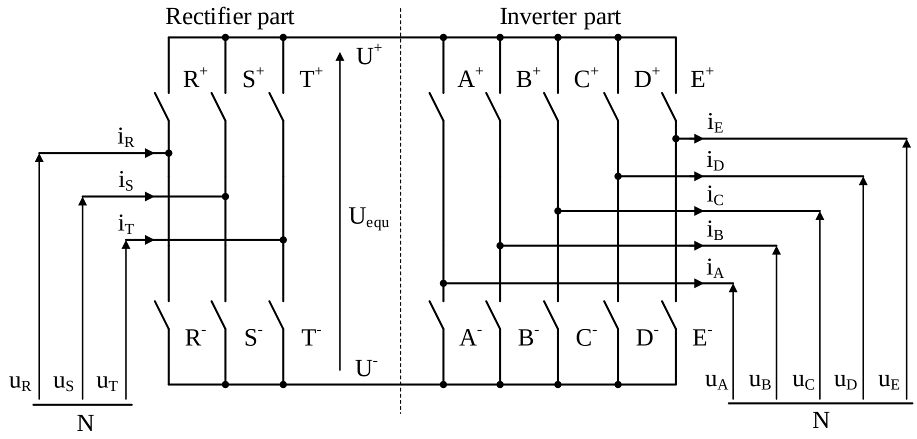

Fig. 2. Schematic model of indirect [3x5] MCS with fictitious interlink with voltage U_{equ}.

The input voltages of the system can be defined as time variable functions

$$u_R = U_R \sin(\omega_0 t),$$

$$u_S = U_S \sin\left(\omega_0 t - \frac{2\pi}{3}\right),$$

$$u_T = U_T \sin\left(\omega_0 t + \frac{2\pi}{3}\right),$$

where U_R, U_S, U_T mean maximal values of the functions and ω_0 is input angle frequency.

Our goal is to calculate output currents i_A, i_B, i_C, i_D, i_E under linear or non-linear passive and active loads.

2 Fictitious interlink and output voltages of matrix converter

The equivalent fictitious interlink voltage function $u_{equ}(t)$ can be calculated as difference of two potentials U^+ and U^- (see fig. 2)

$$u_{equ}(t) = U^+(t) - U^-(t).$$

Difference of U^+ and U^- potentials depends on the maximal and minimal values of input phase voltages u_R, u_S, u_T

$$\begin{pmatrix} U^+(t) \\ U^-(t) \end{pmatrix} = \begin{pmatrix} R^+ & S^+ & T^+ \\ R^- & S^- & T^- \end{pmatrix} \begin{pmatrix} u_R(t) \\ u_S(t) \\ u_T(t) \end{pmatrix},$$

where R$^+$... T^- are logical variables for the maximal and minimal input phase voltages.

Then output voltages u_A, u_B, u_C, u_D, u_E of the system from fig. 2 will be [4]

$$\begin{pmatrix} u_A(t) \\ u_B(t) \\ u_C(t) \\ u_D(t) \\ u_E(t) \end{pmatrix} = \begin{pmatrix} u_{m1}(t), & 1 - u_{m1}(t) \\ u_{m2}(t), & 1 - u_{m2}(t) \\ u_{m3}(t), & 1 - u_{m3}(t) \\ u_{m4}(t), & 1 - u_{m4}(t) \\ u_{m5}(t), & 1 - u_{m5}(t) \end{pmatrix} \begin{pmatrix} U^+(t) \\ U^-(t) \end{pmatrix},$$

where modulation voltages

$$u_{mk}(t) = r\cos(\varphi)\sin\left[\omega_0 t - \frac{2\pi}{5}(k-1)\right] + \frac{1}{2}$$

for $k = 1,\ldots, 5$, $r = \frac{\cos(\varphi-\pi/3)}{\cos(\varphi)}$ and $\varphi = \omega t|_{\mathrm{mod}(\frac{\pi}{3})} - \frac{\pi}{6}$ as it is explained in detail in [4].

3 Modelling of the MCS system with linear passive and active loads

Example 1, considering linear passive resistive-inductive (RL) load

Voltage equation can be expressed using Clarke transform as complex time function in vector form for three-phase system [5-6]

$$\boldsymbol{u}_3(t) = \frac{2}{3}[u_A(t)\boldsymbol{a}^0 + u_B(t)\boldsymbol{a}^1 + u_C(t)\boldsymbol{a}^2]$$

where $\boldsymbol{a}$ is the unit vector $\boldsymbol{a} = \exp\left(j\frac{2\pi}{3}\right) = e^{j\frac{2\pi}{3}}$ and $\frac{2}{3}$ is transform constant for three-phase system.

Thus, in our case, for five-phase system

$$\boldsymbol{u}_5(t) = \frac{2}{5}[u_A(t)\boldsymbol{a}^0 + u_B(t)\boldsymbol{a}^1 + u_C(t)\boldsymbol{a}^2 + u_D(t)\boldsymbol{a}^3 + u_E(t)\boldsymbol{a}^4],$$

where $\boldsymbol{a}$ is the unit vector $\boldsymbol{a} = \exp\left(j\frac{2\pi}{5}\right) = e^{j\frac{2\pi}{5}}$ and $\frac{2}{5}$ is transform constant for five-phase system. As complex time function the voltage $\boldsymbol{u}$ is presented as follows

$$\boldsymbol{u}(t) = \mathrm{Re}\{\boldsymbol{u}(t)\} + j\mathrm{Im}\{\boldsymbol{u}(t)\} = u_\alpha(t) + ju_\beta(t),$$

and it can be decomposed after Clarke transform into three scalar functions

$$u_{5,\alpha}(t) = \frac{2}{5}\left[u_A(t) + u_B(t)\cos\frac{2\pi}{5} + u_C(t)\cos\frac{2\pi}{5} + u_D(t)\cos\frac{2\pi}{5} + u_E(t)\cos\frac{2\pi}{5}\right]$$

$$u_{5,\beta}(t) = \frac{2}{5}\left[u_B(t)\sin\frac{2\pi}{5} + u_C(t)\sin\frac{2\pi}{5} + u_D(t)\sin\frac{2\pi}{5} + u_E(t)\sin\frac{2\pi}{5}\right]$$

$$u_{5,0}(t) = \frac{1}{5}\left[u_B(t)\sin\frac{2\pi}{5} + u_C(t)\sin\frac{2\pi}{5} + u_D(t)\sin\frac{2\pi}{5} + u_E(t)\sin\frac{2\pi}{5}\right] = 0$$

for symmetrical system.

Then, considering the same *RL* load of *α,β* phases

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} i_\alpha(t) \\ i_\beta(t) \end{pmatrix} = \boldsymbol{A}\begin{pmatrix} i_\alpha(t) \\ i_\beta(t) \end{pmatrix} + \boldsymbol{B}\begin{pmatrix} u_{5,\alpha}(t) \\ u_{5,\beta}(t) \end{pmatrix}, \tag{1}$$

where $\boldsymbol{A}, \boldsymbol{B}$ are stationary matrices of the system under *RL* load

$$\boldsymbol{A} = \begin{pmatrix} -\frac{R}{L}, & 0 \\ 0, & -\frac{R}{L} \end{pmatrix}; \quad \boldsymbol{B} = \begin{pmatrix} -\frac{1}{L}, & 0 \\ 0, & -\frac{1}{L} \end{pmatrix}.$$

By time discretization of (1) with the step Δ creating discrete dynamical model [7]

$$\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}_{k+1} = \boldsymbol{F}_\Delta \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}_k + \boldsymbol{G}_\Delta \begin{pmatrix} u_{5,\alpha} \\ u_{5,\beta} \end{pmatrix}_k$$

for $k \in< 0, \infty)$ where k is integer.

It is possible to get the matrices $\boldsymbol{F}_\Delta$ and $\boldsymbol{G}_\Delta$ using some numerical method, e.g.

$$\boldsymbol{F}_\Delta = (\mathbf{1} - \Delta \boldsymbol{A}) ; \quad \boldsymbol{G}_\Delta = \Delta \boldsymbol{B}$$

where the integration step Δ due to possible numerical instability must be sufficiently small, in any case, shorter than the lowest time constant $\frac{L}{R}$ of the MCS system, and also as a step of the sequence of voltage pulses $\frac{T}{5}$, [7-8].

Using modified method for numerical solution [10-12], it is possible to get the result at any time discrete instant k

$$\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}_k = \boldsymbol{F}_\Delta{}^k \begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix}_0 + \boldsymbol{G}_\Delta \sum_{l=0}^{k-1} \boldsymbol{F}_\Delta{}^l \begin{pmatrix} u_{5,\alpha} \\ u_{5,\beta} \end{pmatrix}_{k-(l+1)}$$

for $l \in< k-1, \infty)$ where l is integer.

Real current of the A-phase i_A is equal i_α. Other phase currents i_B, i_C, i_D, i_E can be obtained using inverse Clarke transform $i_{\alpha,\beta,0} \leftrightarrow i_A, i_B, i_C, i_D, i_E$. In steady-state, the other phase currents are shifted against previous one by electric angle $\frac{2\pi}{5}$. Time waveforms of phase voltage and current are shown in fig. 3.

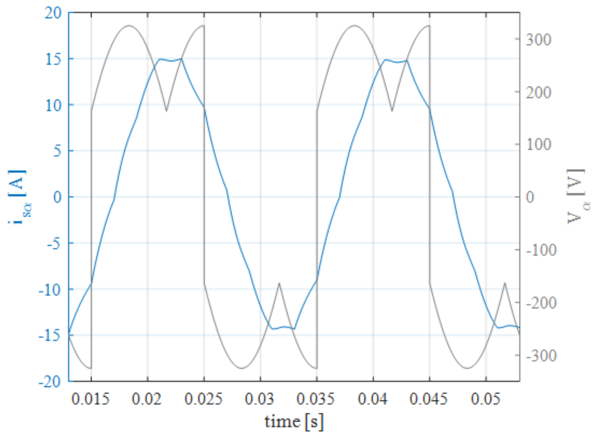

Fig. 3. Phase voltage and current time waveforms of [3x5] matrix converter system at steady-state operation.

Note, that by using this method matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ should be stationary, and time-independent ones. It practically means that elements of these matrices should be constants.

Example 2, considering nonlinear active motoric load - five-phase induction motor

Now, we concentrate our attention to the problems of stator quantities of the motor calculation. It can be expressed using Clarke transform as in the previous example, i.e.

$$\boldsymbol{u}_s(t) = \frac{2}{5}\left[u_A(t) + u_B(t)e^{j\frac{2\pi}{5}} + u_C(t)e^{j\frac{4\pi}{5}} + u_D(t)e^{j\frac{6\pi}{5}} + u_E(t)e^{j\frac{8\pi}{5}}\right],$$

where the voltage is expressed in a component form as

$$\boldsymbol{u}_s(t) = u_{s\alpha}(t) + ju_{s\beta}(t).$$

Then the vectors of stator and rotor currents are possible to be obtained from a dynamic model of the motor [9]

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} i_{s,\alpha}(t) \\ i_{s,\beta}(t) \\ i_{r,\alpha}(t) \\ i_{r,\beta}(t) \end{pmatrix} = \boldsymbol{A}_{non}\begin{pmatrix} i_{s,\alpha}(t) \\ i_{s,\beta}(t) \\ i_{r,\alpha}(t) \\ i_{r,\beta}(t) \end{pmatrix} + \boldsymbol{B}_{non}\begin{pmatrix} u_{5,\alpha}(t) \\ u_{5,\beta}(t) \\ 0 \\ 0 \end{pmatrix} \tag{2}$$

and torque equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\omega_m = \frac{T_{elmg}(t) - T_{load}}{J_m}. \tag{3}$$

Matrices $\boldsymbol{A}_{non}$, $\boldsymbol{B}_{non}$ are in this case non-stationary, they comprise a state-variable ω (see appendix), and $T_{elmg}(t)$ is electro-magnetic torque, T_{load} is load torque, J_m is moment of inertia. System of differential equations (2)-(3) is nonlinear one due to nonlinear functions such as multiplications of state-variables. So, the method mentioned above can not be used.

Linearization of the system equations (2)-(3) can be done using exciting fictitious functions: non-stationary elements of the matrices $\boldsymbol{A}_{non}$, $\boldsymbol{B}_{non}$ are withdrawn from them, and fictisious exciting functions are created. Then

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} i_{s,\alpha}(t) \\ i_{s,\beta}(t) \\ i_{r,\alpha}(t) \\ i_{r,\beta}(t) \end{pmatrix} = \boldsymbol{A}_{lin}\begin{pmatrix} i_{s,\alpha}(t) \\ i_{s,\beta}(t) \\ i_{r,\alpha}(t) \\ i_{r,\beta}(t) \end{pmatrix} + \boldsymbol{B}_{lin}\begin{pmatrix} u_{5,\alpha}(t) \\ u_{5,\beta}(t) \\ f(\omega, i) \\ f(\omega, i) \end{pmatrix}.$$

Now, we can generate new, linearized system of differential equations, and after time discretization the numerical solution can be used:

$$\begin{pmatrix} i_{s,\alpha}(t) \\ i_{s,\beta}(t) \\ i_{r,\alpha}(t) \\ i_{r,\beta}(t) \end{pmatrix}_{k+1} = \boldsymbol{F}_{lin}\begin{pmatrix} i_{s,\alpha}(t) \\ i_{s,\beta}(t) \\ i_{r,\alpha}(t) \\ i_{r,\beta}(t) \end{pmatrix}_{k} + \boldsymbol{G}_{lin}\begin{pmatrix} u_{5,\alpha}(t) \\ u_{5,\beta}(t) \\ f(\omega, i) \\ f(\omega, i) \end{pmatrix}_{k}.$$

Note, that linearized system with $\boldsymbol{A}_{lin}$, $\boldsymbol{B}_{lin}$ should be checked for stability and controllability before numerical computations. Time waveforms of phase voltage and current are shown in fig. 4.

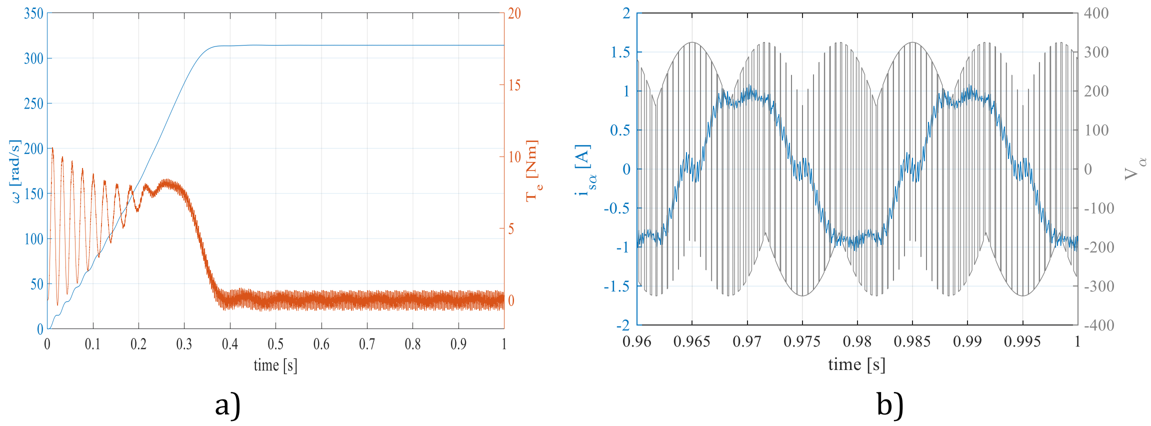

Fig. 4. Start-up of the motor which is fed by [3x5] matrix converter system for speed and torque (a), detailed waveforms of phase voltage and current at steady-state (b) gained by computer simulation.

7 Conclusion

The method introduced in the paper presents good mean for matrix converter system solution under periodic non-harmonic impulse supply. The method makes possible to obtain the instantaneous state of the system if the system of matrices are stationary ones. Otherwise, one can use approximated approach. So, a new modified method of solution makes possible to determine the state of the system instantaneously at any discrete time instant. Worked-out results have confirmed that theoretical assumptions and verifying computer simulations are in good agreement.

Appendix

$$\boldsymbol{A}_{non} = \begin{pmatrix} -\frac{R_r}{L_r} & \omega_r & M\frac{R_r}{L_r} & 0 \\ -\omega_r & -\frac{R_r}{L_r} & 0 & M\frac{R_r}{L_r} \\ \frac{MR_r}{\sigma L_s L_r^2} & \frac{M}{\sigma L_s L_r}(\omega_s - \omega_r) & -\frac{R_s}{\sigma L_s} & \omega_s \\ \frac{M}{\sigma L_s L_r}(\omega_s - \omega_r) & \frac{MR_r}{\sigma L_s L_r^2} & -\omega_s & -\frac{R_s}{\sigma L_s} \end{pmatrix};$$

$$\boldsymbol{B}_{non} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{M}{\sigma L_s L_r} & 0 & \frac{1}{\sigma L_s} & 0 \\ 0 & -\frac{M}{\sigma L_s L_r} & 0 & \frac{1}{\sigma L_s} \end{pmatrix},$$

where elements of matrices are derived from motor parameters [9], except for the angular velocity ω_s or ω_r, respectively, which is one of the state variables.

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