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NETWORK FLOWS AS A TOOL FOR SOLVING LOCATION-SCHEDULING PROBLEM TO OPTIMIZE CHARGING INFRASTRUCTURE FOR ELECTRIC VEHICLES

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Abstract. We propose an MIP model to design a private charging infrastructure for a fleet of electric vehicles operating in large urban areas. We derive an IP problem and we show that it is equivalent to a network flow model. We illustrate that the flows in networks can be used in the study of properties of location-scheduling problems related to the design of charging infrastructure.

Keywords: electric vehicles, charging infrastructure, mixed integer programming, flows in networks

Mathematics subject classification: Primary 90B06; Secondary 94C15, 68R10

We propose mixed integer programming (MIP) and integer programming (IP) models to design a private charging infrastructure for a fleet of electric vehicles operating in large urban areas. Examples of such a fleet include taxicabs and small vans used in city logistics or with shared vehicles. The fleet is composed of vehicles equipped with an internal combustion engine, but the operator is wishing to replace them with fully electric vehicles in the near future. Hence it is required to design a private network of charging stations that will be specifically adjusted to the operation of the fleet. We use GPS traces to characterize actual travel patterns of individual vehicles. We formulate location-scheduling optimization models that determine the maximum number of vehicles that can be recharged without affecting their routes and parking behaviour (MIP model) or minimize the number of charging stations under the condition that all vehicles can be recharged (IP model). The MIP model assumes that all vehicles possess complete information about all other vehicles. To study the role of available information, we evaluate the resulting designs considering uncoordinated charging when vehicle drivers know only the actual occupation of charging points at the time when they are choosing a charging station. Tests show a gap between optimization and uncoordinated charging. We suggest a network flow model that allows us to study the gap and properties of the location-scheduling problem. We also show that the network flow model can be formulated as the IP model.

1 Introduction

Advances in battery technologies and decreasing prices of electric vehicles (EVs) stimulate more interest in converting large fleets of vehicles into electric. Benefits could be considerable due to high utilization of such vehicles. Recently, mathematical optimization has been often proposed to support the design of charging infrastructure. The special class of models was developed to cover trajectories of vehicles [1, 6]. This approach is applicable in the design of the charging infrastructure along motorways to cover long distance trips. The extension of this approach that considers multiple paths connecting origins and destinations of trips was proposed in [8].

We assume that the operation of the fleet should remain unaffected, i.e. for the recharging of batteries only parking events are used. Here we propose an extension of the approach [4, 5], where optimization can select vehicles that are successfully converted to electric, we consider charging points with different charging speeds and evaluate the role of available information when choosing a charging point.

We propose a mixed integer programming (MIP) model of the problem (Section 2). To investigate the role of available information for the MIP model, we introduce a procedure for uncoordinate charging of vehicles (Section 3). The results of numerical experiments can be found in Section 4. It follows from the experiments that the MIP model represents minimal requirements for charging infrastructure. We suggested an integer programming (IP) model (Section 5) and equivalent flow model (Section 6) to analyse further behaviour of the proposed problem.

2 Mathematical model

The fleet of vehicles is represented by the set C and each vehicle is equipped with a battery of capacity β (for simplicity, we suppose that all vehicles have the same capacity that determines the maximum driving range). The set of candidate locations to locate charging stations is denoted by I. We discretise the time by dividing it into the set of non-overlapping intervals T of equal size. For each vehicle $c \in C$, we extract from GPS data an ordered sequence of parking events R_c and N_{cr} denotes the list of time intervals $t \in T$ that have an overlap with the parking event $r \in R_c$. To simplify notation, for each vehicle we add to R_c the fictional introductory parking event 0 and fictional terminal parking event r_c. The set of charging speeds is denoted by S (here we consider $|S| = 3$). The price to build a charging point of type $s \in S$ is p_s and γ denotes large positive constant. From GPS data we extract driving distances and we estimate the battery capacity consumption when vehicle $c \in C$ drives towards stop $r \in R_c$ as $u_{rc} \geq 0$. $a_{ct} \in \langle 0, 1 \rangle$ is the fraction of the time interval $t \in T$ when vehicle $c \in C$ is parking. To simplify the description of the model, we define $B_{itc} \in \{0, 1\}$, where $B_{itc} = 1$ if vehicle $c \in C$ is parking closer than the distance ρ_{max} from location $i \in I$ during the time interval $t \in T$, and $B_{itc} = 0$ otherwise.

Decisions are described by the following variables:

- $w_{is} \in Z^+$ represents the number of charging points of charging speed $s \in S$ allocated to station $i \in I$,
- $z_c \in \{0, 1\}$, when vehicle $c \in C$ exceeds driving range and thus cannot be successfully converted to electric $z_c = 1$, otherwise $z_c = 0$,
- $x_{cts} \in \{0, 1\}$, if vehicle $c \in C$ is being charged with speed $s \in S$ during the time interval $t \in T$, then $x_{cts} = 1$, and otherwise $x_{cts} = 0$,

- $q_{cts} \geq 0$ represents the part of interval $t \in T$ when vehicle $c \in C$ is being charged with speed $s \in S$,
- $d_{cr} \geq 0$ corresponds to the state of charge of the vehicle $c \in C$ at the beginning of the parking event $r \in R_c$.

We consider the objective function minimizing the number of vehicles that exceed driving range, i.e.:

$$\begin{aligned}
\text{Minimize} \sum_{c\in C} z_c & & (1)\\
\text{subject to} \sum_{c\in C} B_{itc}x_{cts} \leq w_{is} & \quad \text{for } i \in I, t \in T, s \in S & (2)\\
d_{c0} \leq 0.5\beta + z_c\gamma & \quad \text{for } c \in C & (3)\\
d_{c,r_c} + z_c\gamma \geq 0.5\beta & \quad \text{for } c \in C & (4)\\
d_{cr} + \sum_{s\in S, t\in N_{cr}} q_{cts}s \leq \beta & \quad \text{for } c \in C, r \in R_c - \{0\} & (5)\\
d_{cr} \leq d_{c,r-1} - u_{cr} + \sum_{s\in S, t\in N_{c,r-1}} q_{cts}s + z_c\gamma & \quad \text{for } c \in C, r \in R_c - \{0\} & (6)\\
\sum_{s\in S} x_{cts} \leq 1 & \quad \text{for } c \in C, t \in T, s \in S & (7)\\
q_{cts} \leq x_{cts}a_{ct} & \quad \text{for } c \in C, t \in T, s \in S & (8)\\
\sum_{i\in I, s\in S} w_{is}p_s \leq G & & (9)
\end{aligned}$$

Constraints (2) make sure that at each station and each time interval we cannot use more charging points of a given type than available. Constraints (3) (constraints (4)) ensure that initial (final) state of charge of each vehicle is not more (less) than 50% of the battery capacity. Constraints (5) make sure that the battery capacity cannot be exceeded. Constraints (6) ensure the contiguity in charging and discharging of vehicles. Constraints (7) make sure that a vehicle cannot be charged simultaneously at more than one speed. Constraints (8) ensure that a vehicle is being charged within the time interval $t \in T$ only when vehicle is parking and $x_{cts} = 1$. Finally, constraint (9) keeps the costs to set up the charging infrastructure to be less or equal than the budget limit G.

3 Evaluating the role of available information

In previous section, we supposed that complete information about requests of customers has been available. But reality is often different, hence, to evaluate designs of charging stations found by the model, we consider approach, which works with uncoordinated charging strategy where the users know only the occupancy of charging stations when they chose a charging station, but have no information on when the charging stations are freed. The set of charging stations found by model (1)-(9) is denoted by $\overline{I}$. K_i is the set of charging points placed at station $i \in \overline{I}$ (we suppose that $0 \notin K_i$ and we set $k_{max} = 0$). $R = \cup_{c\in C}R_c$ is the set of possible charging events, P is the set of movements of vehicle, and $E = R \cup P$ is the set of all events. The charging speed of point $k \in K_i$ is denoted as s_k. If $e \in P$, then $l(e)$ is the driving distance.Thus, we assume that vehicles are in one of the two modes, either parking or driving. Vehicle associated with event $e \in E$ is denoted by $c(e)$. The start time and the end time of the event $e \in E$ are denoted by $b(e)$ and $z(e)$, respectively. Level of the battery of vehicle $c \in C$ is d_c. Value ρ_{max} is the maximum acceptable proximity of a vehicle from a station to make the charging still possible, and T_k is the end time of the most recent charging event at point $k \in K_i$, for $i \in \overline{I}$.

3.1 Evaluation procedure

We evaluate the performance of charging infrastructure by running the following algorithm:
Step 1: (Initialization)
For $c \in C$ set $d_c = \beta/2$. Order events from E ascendingly with respect to $b(e)$. Set $T_k = 0$ for $k \in K_i$ and $i \in \overline{I}$.
Step 2: (Event list processing)
For each $e \in E$ do:

> If $e \in R$, then order the set $\overline{I}$ descendingly with respect to the sum of speeds s_k over charging points $k \in K_i$ that are free in time $b(e)$.
> For $i \in \overline{I}$ do:
>
> > If the distance of vehicle $c(e)$ at time $b(e)$ from station i is less than ρ_{max}, then process the charging event following an uncoordinated charging strategy.
>
> If $e \in P$, then set $d_{c(e)} = d_{c(e)} - l(e)$.

Step 3: (Evaluation)
Evaluate feasibility of all vehicles for $c \in C$. If $d_c \geq 0$ all the time during the run of the algorithm and $d_c \geq \beta/2$ at the time when the algorithm is terminated, then $c \in C$ is feasible.
Output of the procedure are decisions about feasibility of each vehicle.

Strategy of uncoordinated charging

We suppose that drivers have only information about the occupancy of charging points and unplug their vehicles at the time of departure for the next trip.

Find $k_{max} \in argmax(s_k)$[1], where $k \in K_i$ and $T_k \leq b(e)$.
If $k_{max} \neq 0$:

> Set $d_{c(e)} = \min\{\beta, d_{c(e)} + (z(e) - b(e))s_{k_{max}}\}$. Set $T_{k_{max}} = z(e)$, $k_{max} = 0$ and continue with step 2, and process the next event.

4 Numerical experiments

In the case study, we consider the fleet C of 500 taxicabs operating in the area of great Stockholm, in Sweden, whose positions were recorded from 01/May/2014 until 14/May/2014. Each vehicle reported on average every 90 seconds its id, GPS position, time-stamp and information whether it is hired or not.[2] When solving optimization model we split the time period covered by the data into the set T of 1344 time intervals, each of length 15 min. We used GPS data to deduce two sets of candidate locations (with $|I| = 5$ and $|I| = 20$), the sets R_c, N_{cr} and P using the procedure described in [5], while setting parameters $V_{max} = 0.1$ m/s and $M = 150$. To test the proposed approach, we performed numerical experiments with the following values of parameters: the driving range of all vehicles was set to $\beta = 300$ km and vehicles can be charged only if they park closer than $\rho_{max} = 500$

[1] If $argmax$ returns a non-empty set, then k_{max} can be an arbitrary element from this set.
[2] Data have been provided by M. Cebecauer from KTH Royal Institute of Technology in Stockholm.

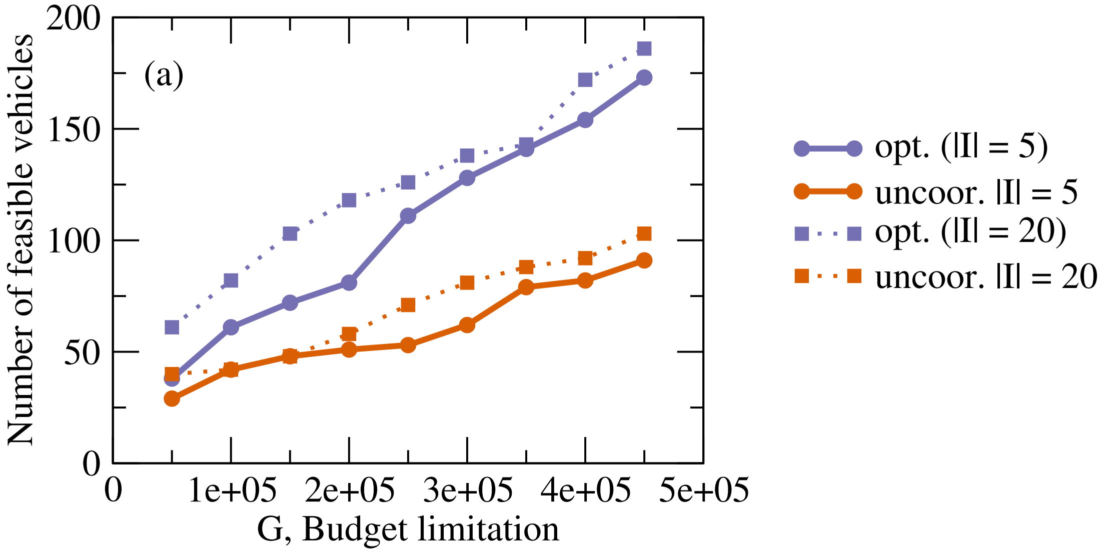

Fig. 1. The number of feasible vehicles as a function of budget limit G obtained by solving the optimization model and applying the evalution procedure using uncoordinated charging strategy, each for $|I| = 5$ and $|I| = 20$. The values achieved for the largest value of G remain further unchanged when G is increased.

metres from the charging station. We considered three types of charging points characterized by the set of charging speeds $S = \{5.3, 21.3, 74.6\}$ km/hour (we call them slow, medium and fast) and the corresponding set of installation costs $p = \{500, 2\,500, 25\,000\}$ USD.

For $|I| = 5$ ($|I| = 20$) and unlimited budget G, we obtained 8 (2) slow, 15 (5) medium and 16 (21) fast charging points. In Figure 1, we show the dependency between the number of feasible vehicles obtained from mathematical model and obtained by running the evaluation procedure of uncoordinated charging as a function of the budget limit G. Gap between results obtained by the optimization model and unccordinated charging strategy gives an idea about the role of available information. The number of feasible vehicles that can be turned into electric is systematically significantly higher for the optimization model. The optimization model assumes availability of perfect information about other vehicles, when drivers choose a charging stations. While in the case of uncoordinated charging strategy, information about other vehicles is limited to the occupancy of charging stations only.

5 Discrete model

Our results indicate that this approach can be used to estimate the minimal requirements to set up the charging infrastructure. When we want to better explain the gap between optimization and uncoordinate charging procedure, we need to understand the behaviour of the studied problem. This is the reason, why we simplify and suggest an IP model, which has very similar structure, but it is posed differently. We do not look for the maximum number of vehicles that can be served by the limited charging infrastructure, but for a given number of vehicles we search for a minimal infrastructure that can serve all vehicles. We work with the following variables: $y_i \in \{0, 1\}$, where $y_i = 1$ when a charging station is placed at the location $i \in I$ and $y_i = 0$ otherwise; $x_{ct} \in \{0, 1\}$, where $x_{ct} = 1$ when vehicle $c \in C$ is charged during the time interval $t \in T$ and $x_{ct} = 0$ otherwise; and $d_{cr} \in Z_0^+$ is the distance that the vehicle $c \in C$ is able to drive at the beginning of the parking event $r \in R_c$. Constant β is the capacity of battery, m is the maximum number of charging points. We estimate the

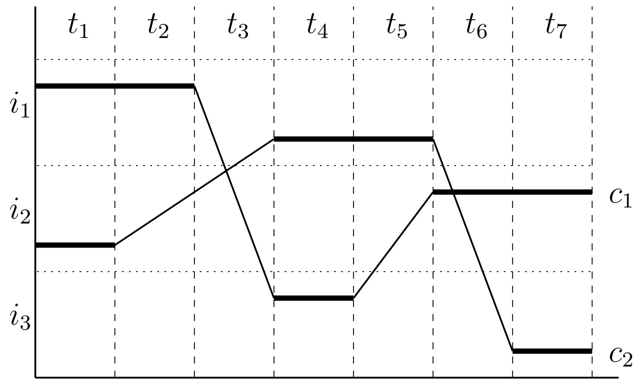

Fig. 2. Diagram illustraiting parking events that represent charging opportunities (thick horizontal lines) and transfer of vehicles between charging stations (thin diagonal lines).

battery capacity consumption when vehicle $c \in C$ drives towards stop $r \in R_c$ as $u_{rc} \in Z^+$.

$$\text{Minimize} \sum_{i \in I} y_i \tag{10}$$

$$\text{subject to} \sum_{c \in C} B_{itc} x_{ct} \leq m y_i \qquad \text{for } i \in I, t \in T \tag{11}$$

$$d_{cr} + \sum_{t \in N_{c,r}} x_{ct} \leq \beta \qquad \text{for } c \in C, r \in R_c \cup \{r_c\} \tag{12}$$

$$d_{cr} \leq d_{c,r-1} - u_{cr} + \sum_{t \in N_{c,r-1}} x_{ct} \qquad \text{for } c \in C, r \in R_c \cup \{r_c\} \tag{13}$$

In the objective function (10), we minimize the number of charging stations. Constraints (11) ensure that we cannot use more charging points at each location and in each time interval than available. Constraints (12) ensure that battery capacity is never exceeded and constraints (13) ensure the contiguity of charging and discharging of batteries.

6 Flow model

In [2], we introduced a flow model approximating this problem. In this section we amend the model to be equivalent with IP model. We construct a network G. The set of vertices of G is the union of the sets:
$V_1 = \{v_{it} : i \in C, t \in T\}$, where v_{it} represents the vehicle i in time t,
$V_2 = \{u_{it} : i \in I, t \in T\} \cup \{w_{it} : i \in I, t \in T\}$, where u_{it} and w_{it} represent the candidate location i in time t,
$V_3 = \{s, z, w\}$, where s is the source, z is the sink and w an additional vertex of the network.

The set of edges of G is the union of the sets:
$E_1 = \{(s, v_{i,1}) : i \in C\}$,
$E_2 = \{(s, u_{i,1}) : i \in I\}$,
$E_3 = \{(v_{i,t-1}, v_{i,t}) : i \in C, t \in T - \{1\}\} \cup \{(v_{i,n}, z) : i \in C, n = \max(T)\}$,
$E_4 = \{(u_{i,t-1}, u_{i,t}) : i \in I, t \in T - \{1\}\} \cup \{(u_{i,n}, z) : i \in I, n = \max(T)\}$,
$E_5 = \{(u_{i,t}, w_{i,t}) : i \in I, t \in T\} \cup \{(w_{i,t}, u_{i,t+1}) : i \in I, t \in T - \{n\}\} \cup \{(w_{i,n}, z) : i \in I\}$,

E_1	$\langle \beta, \beta \rangle$	E_5	$\langle 0, m \rangle$
E_2	$\langle 0, \lvert T \rvert \rangle$	E_6	$\langle 0, 1 \rangle$
E_3	$\langle 0, \beta \rangle$	E_7	$\langle 1, 1 \rangle$
E_4	$\langle 0, \lvert T \rvert \rangle$	E_8	$\langle 0, \infty)$

Tab. 1. Lower and upper bounds for the flows on edges.

$E_6 = \{(w_{i,t}, v_{j,t}) : i \in I, t \in T, j \in C, B_{i,t,j} = 1\}$,
$E_7 = \{(v_{j,t}, w) : j \in C, t \in T, \sum_{i \in I}\} B_{i,t,j} = 0$,
$E_8 = \{(w, z)\}$.
Every edge has assigned to it a lower and upper bound for the flow, which are represented by the interval $\langle l, u \rangle$. These bounds are given in Table 1. It is possible to show that the optimal solution of IP model is equivalent to the feasible flow in G with the minimum number of non-zero flows on the edges of the set E_2. We will call this problem: a sparse feasible flow. If the flow on the edge $(s, u_{i,1}) \in E_2$ is non-zero, then we place a charging point in location i. Problems of finding a feasible flow in the network with lower bounds on edges and its minimisation are solvable in polynomial time [3]. We suppose that the problem to find a sparse feasible flow in network is NP-hard problem. Hence we suggest heuristic method to solve it:
Description of the algorithm. We construct a network G' (it is subnetwork of G):
Its vertex set is $V_1' \cup V_2' \cup V_3'$, where
$V_1' = \{v_{i,t} : i \in C, t \in T\} \cup \{z\}$,
$V_2' = \{u_{i,t} : i \in I, t \in T\} \cup \{w_{i,t} : i \in I, t \in T\}$.
The edge set is the union of the sets:
$E_1' = \{(v_{i,t-1}, v_{i,t}) : i \in C, t \in T - \{1\}\} \cup \{(v_{i,n}, z) : i \in C\}$,
$E_2' = \{(u_{i,t-1}, u_{i,t}) : i \in I, t \in T - \{1\}\}$,
$E_3' = \{(u_{i,t}, w_{i,t}) : i \in I, t \in T\} \cup \{(w_{i,t}, u_{i,t+1}) : i \in I, t \in T - \{n\}\} \cup \{(w_{i,n}, z) : i \in I\}$,
$E_4' = \{(w_{i,t}, v_{j,t}) : i \in I, t \in T, j \in C, B_{i,t,j} = 1\}$,
$E_5' = \{(u_{i,n}, z) : i \in I, n = \max(T)\}$.
Lower and upper bounds of the edges are taken from G.

Algorithm.
Input is the network G with feasible flow x and subnetwork G'.
For each pair of vertices $u_{i,2}, u_{j,2} \in V_2$ such that $0 < x(s, u_{i,2}) \leq x(s, u_{j,2}) < |T|$ **do**:

while there is an augmenting path $P(j, i)$ with reserve r from $u_{j,2}$ to $u_{i,2}$ in G' **do**:

add the edges $(s, u_{j,2})$ and $(s, u_{i,2})$ to $P(j, i)$ to form a (non-oriented) cycle
$C = (s, u_{j,2}, \ldots, u_{i,2}, s)$,
change the flow x in C as follows:
if (u, v) is the forward edge in C, **then** $x(u, v) = x(u, v) + r$,
if (u, v) is the reverse edge in C, **then** $x(u, v) = x(u, v) - r$,

process another pair of vertices.

Output is a new feasible flow.

A polynomial algorithm for finding an augmenting path in network is presented in [7].
This approach does not guarantee that given solution is optimal. We aim to test the algorithm and

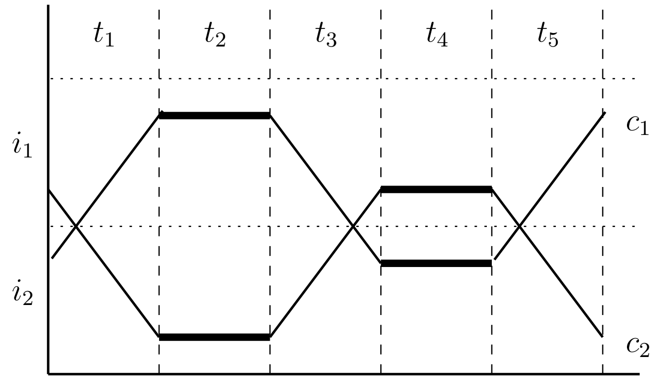

Fig. 3. Diagram from Example 1.

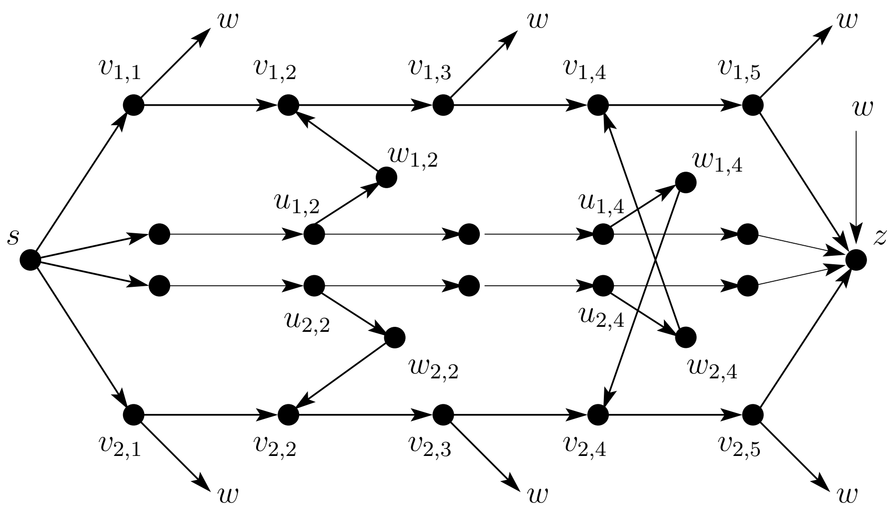

Fig. 4. Network G from Example 1.

study its properties in our future works.

Example 1.
To illustrate, how the graph G can be constructed, we consider situation with two vehicles, two candidate locations and five time intervals as can be seen in Figures 3 and 4. Existence of a path $u_{k,j} \to w_{k,j} \to v_{i,j}$ means that the vehicle i is parking in location k at the time interval j.

What is the cause of the gap in the number of feasible vehicles obtained from the optimization problem and simulation procedure?

We suppose that the main reason is in the available information. When we use an optimization algorithm, we work with the whole network G. However, in the simulation procedure, at time $t \in T$, we only have information about edges which start in vertices $u_{i,t}$. These constraints do not allow us to find an optimal solution by the evaluation procedure. The open questions are: How do we design the charging infrastructure especially in cases when the limited level of information is given? How can we spread the available information (from O-D matrices, probability models) to obtain a lower number of unfeasible vehicles?

7 Conclusions

Our results indicate that optimization is able to serve significantly more vehicles than uncoordinated charging strategy. Being motivated by the goal to explain this differences we introduced simpler IP model and associated flow model. Initial study of model properties helped us to gain intuition about the gap between optimization and simulation of charging. We expect that better understanding of the behaviour of these models could lead to computationally more efficient solving methods.

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References

[1] CHUNG, S., H., KWON, C.: Multi-period planning for electric car charging station locations: A case of Korean Expressways. *European Journal of Operational Research, 242(2)*, pp. 677–687, 2015.

[2] CZIMMERMANN, P., BUZNA, Ľ., KOHÁNI, M.: The Design of Charging Infrastructure for Electric Vehicles and Its Properties. In *Proceedings of the 15th International Symposium on Operational Research*, 2017.

[3] DANTZIG, G., B., FULKERSON, D., R.: Minimizing the number of tankers to meet a fixed schedule. *Naval Res. Logist. Quart., 1*, pp. 217–222, 1954.

[4] KOHÁNI, M., CZIMMERMANN, P., VÁŇA, M., CEBECAUER, M., BUZNA, Ľ.: Designing Charging Infrastructure for a Fleet of Electric Vehicles Operating in Large Urban Areas. In *Proceedings of the 6th International Conference on Operations Research and Enterprise Systems - Volume 1:ICORES*, pp. 360–368, 2017.

[5] KOHÁNI, M., CZIMMERMANN, P., VÁŇA, M., CEBECAUER, M., BUZNA, Ľ.: Location-scheduling optimization problem to design private charging infrastructure for electric vehicles. In *Operations Research and Enterprise Systems, ICORES 2017, Revised Selected Papers.* Communications in Computer and Information Science, Springer. (Accepted for publication.)

[6] MIRHASSANI, S., A., EBRAZI, R.: A Flexible Reformulation of the Refueling Station Location Problem. *Transportation Science, 47(4)*, pp.617–628, 2013.

[7] NEMHAUSER, G., L., WOLSEY, L., A.: Integer and Combinatorial Optimization. *Wiley-Interscience series in discrete mathematics and optimization.* ISBN 0-471-82819-X, pp. 65, 1988.

[8] YILDIZ, B., ARSLAN, O., KARASAN, O., E.: A branch and price approach for routing and refueling station location model. *European Journal of Operational Research, 248(3)*, pp.815–826, 2016.

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