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NOTES TO THE PROBLEMS WITH FIGURES AT SCHOOL MATHEMATICS

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Abstract. The aim of this paper is to analyse the achieved level of the special mathematical competence at Slovak lower secondary schools in recent years. The attention will be given to the analysis of the pupil knowledge in understanding and correct solution of mathematical tasks which we have named *problems with figures*. The identification of problematic thematic units of mathematics at schools is good feedback for the preparation of the future mathematics teachers at the universities.

Keywords: statistical analysis, teaching mathematics, evaluation

Mathematics Subject Classification: 97B10, 97C70, 97D60.

1 Introduction

Currently, people are constantly inundated with new information of all kinds. On the road to knowledge, the development of the pupils' ability to understand and correctly process information from the text implies therefore an increasing importance of the role of schools in this process. Moreover, mathematical education at schools creates a space for understanding the text, which contains charts, diagrams and other forms of images.

In our contribution, we focused on the level of graduates of our secondary schools in the area of understanding and the subsequent correct solution of mathematical tasks of this type, which we have named *problems with figures*. [1]

The analysis was made on the basis of nationwide testing T9 from the last six years. The nationwide testing takes a special place in evaluating of the learning outcomes. Testing T9 is aimed at 15-year-old pupils before leaving the lower secondary school. From 2013 the tested content units included in mathematical part of T9 have been following (in accordance with [7]):

- Numbers, variables and arithmetic operations with numbers,
- Relations, functions, tables, charts,

- Geometry and measurement,
- Combinatorics, probability, statistics, logic, reasoning, evidence.

In the paper we present some of our findings in this area: a) what is the trend in success in solving problems of this type, b) what the problems with figures can "say" about the mathematical education. Finally, we propose some recommendations connected with these problems for higher education of future mathematics teachers.

The low overall success rate in this testing is largely due to poor results of the content unit Geometry and measurement. In more detailed qualitative analysis of T9 tasks, we had focused on geometry tasks, but we were also interested in something else. The text passages in the assignment of some tasks are completed with "figures". In these tasks, the part of the input data or data in options in closed test items are graphically visualized by means of the figures (scheme, graph, table, picture, map). Such group of tasks is at the centre of our interest in the following part of this paper.

2 Problems with figures

At first sight, the problems with figures can be understood as the problems from the content unit Geometry and measurement. But not all "geometric" problems in T9 are with figures and some problems with figures are not geometric and come from other content units (see Fig. 1).

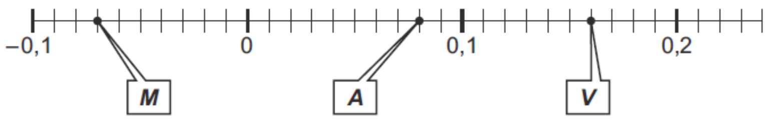

Fig. 1. The figure from the test item 02/2017.
(Numbers M, A, V are shown on the number axis. Calculate $M + A + V$.)

The numbers of the problems with figures in each year of the period 2012–2017 are in the Tab. 1.

Year	2012	2013	2014	2015	2016	2017
Number of problems with figures	8	9	5	5	7	7

Tab. 1. The numbers of the problems with figures in years 2012–2017.
(Total number of the tasks is 20 in each year.)

The most of the problems with figures in T9 focus on the following areas and aims:
- *angles* – calculation of the measure of the marked angles in planar objects,
- *planar objects* – determination of the area or the circuit of a given planar object using formulas or a square grid,

- *spatial objects* – determination of the surface area or the volume of a given spatial object using formulas or determination of the surface area of a composite 3D object composed of identical cubes,
- *number axis* – determination of the position of a given number on the number axis,
- *part and whole* – determination of the part of the whole as a fraction or a percentage and vice-versa,
- *charts* – reading the numerical data from the chart and determination of some numerical characteristics (e. g. average) or truth values of given statements.

3 Quantitative statistical analysis

The statistical analysis we made with the aim to compare the level of the mathematical competence in the last six years in Slovakia. The source of the data was the anonymized database provided by NICEM. It contains information concerning the results of the tests of 242 830 pupils of the ninth grade of lower secondary schools across Slovakia from years 2012–2017. For our analysis, we used a point-based evaluation of all tasks in the tests in the selected period as well as the information on the sex of pupils and the founders of the schools that they attended (state, private, church). Besides the overall evaluation, we special paid attention to the results achieved in the group of problems with figures. The statistical analysis was carried out in the statistical program IBM SPSS and all charts and tables in this section are the outputs of this program.

As it was mentioned above, the number of problems with figures was not the same in each year. Therefore, we analysed the percentage of success in this unit. Initial information can be obtained from the next chart (Fig. 2). We can observe that after the gradual increase of the selected parameter in 2012–2015, a rapid decrease in the success rate of pupils came in 2016. In 2017 the success rate reached the level of 2012. In the next part we selected one sample from the problems with figures from T9 in 2016 with a small successfulness and tried to explain what could cause their unsuccessfulness (apart from reduction of mathematical skills of pupils).

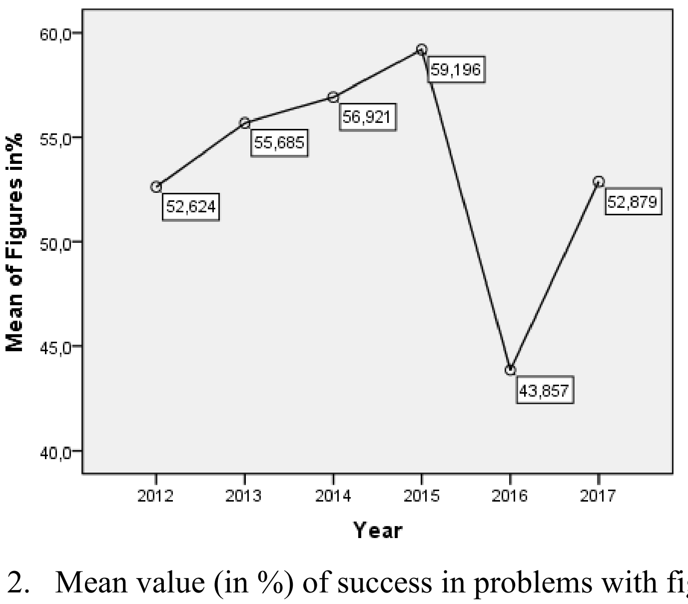

Fig. 2. Mean value (in %) of success in problems with figures.

From the following chart (Fig. 3) we can compare the total success in T9 and the success in the problems with figures. The chart shows that only in years 2014 and 2015 the pupils' success of the problems with figure was higher than their overall success. It looked like a good sign that the figure in the assignment of the task is very important, it helps pupils to process better the data in the assignment, or the illustrative image in the assignment can help them solve the task. However, we can observe, that in the last two years, there has been a significant decline in success in examples with figures.

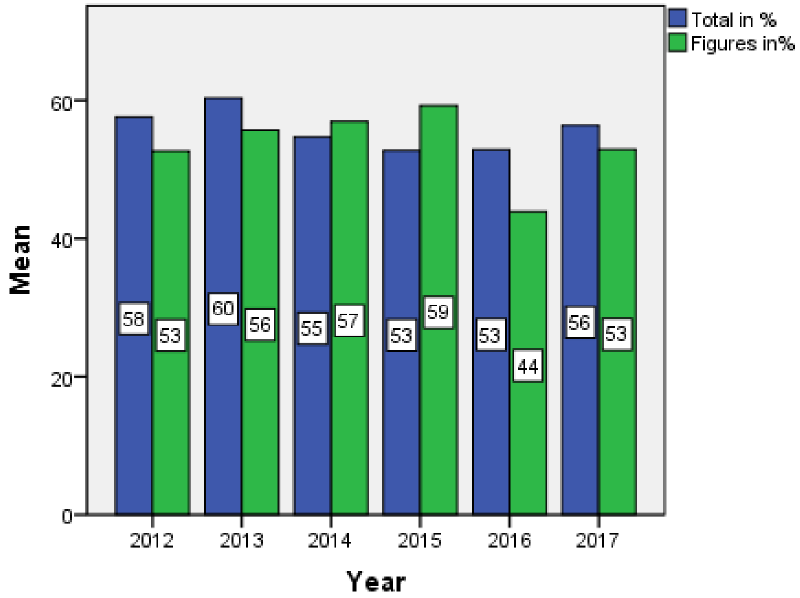

Fig. 3. Mean value (in %) of results in T9 and problems with figures.

In the case of the total success in T9 we observed that the distribution of values is for boys and girls alike. However, for problems with figures, it is a clear significant difference of the distribution of gained points (Tab. 2). Another more detailed analysis has shown that even one of the reference years girls did not exceed the average success of boys.

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of total is the same across categories of sex.	Independent-Samples Mann-Whitney U Test	,311	Retain the null hypothesis.
2	The distribution of figures is the same across categories of sex.	Independent-Samples Mann-Whitney U Test	,000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is ,05.

Tab. 2. Mann-Whitney U test for the distribution of total T9 and of problems with figures across sex.

Subsequently we compared the average successfulness in the problems with figures for each category of the founder. Considering the test results, we can conclude that especially in the last two reporting year, private schools showed significant success in solving the problems with the figures in comparison with state and church schools. We can also say that public schools in any particular year showed the lowest mean value of achieved points in the problems with figures.

4 Selected problems with figures in T9

For a more detailed view of individual task and typology of tasks, it can be noticed that recurring tasks (or recurring type of task) in T9 are characterized by a higher success of the solution (see [2], [4]). As the example, the test item 5/2017 from content unit Geometry and measurement can be mentioned: *The ray o shown in the figure is the axis of the angle β. Calculate the magnitude of the angle δ in degrees.* (Fig. 4)

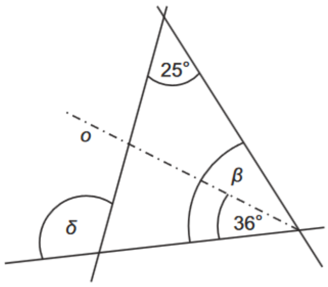

Fig. 4. The triangle (test item 5/2017).

Solution of this task is based on the knowledge of the sum of the internal angles' values in the triangle and the sum of the supplementary angles, and its successfulness was 52,2%. Similar tasks occurred in 2012, 2013, 2014, and 2015. This type of task is included to the typical geometric tasks in school mathematics, it is also used in textbooks for a long time, and the pupils can solve it because it is trained for many times.

On the other side, there are types of tasks which have not usually occurred in T9 (and also in the school mathematics) and pupils often fail in their solutions. The following problem is the example of the non-occurring type of test item with a very low successfulness: *The net of a cuboid is shown in the figure; two dimensions are 11 cm and 4,5 cm. Calculate the third dimension of this cuboid in centimetres.* (Fig. 5)

The success rate of this test item's solution was the second lowest one in 2017 – 29,1%. Its solution is based on right "reading" of the given figure (the net of the cuboid), and one of the possibilities how to solve this problem can be strategy "trial and error". It requires neither a formula nor a special algorithm, only the spatial imagination.[1]

[1] The similar task was occurred in external part of final examination at the secondary school in 2016 (18/2016), but the success rate was higher – 73,3%.

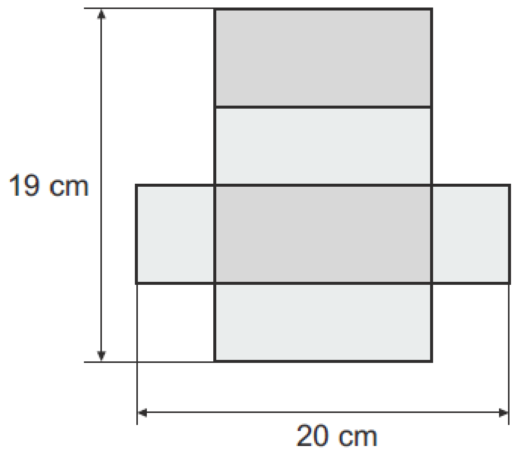

Fig. 5. The net of the given cuboid (test item 09/2017).

During the lessons of the subject Didactics of mathematics, our students solved some types of tasks from T9. The students of mathematics teaching had no problems with the previous problem. But it was problematic for the students of primary teaching. Only 23 % students solved it right (e. g. see Fig. 6), 18 % of them did not solve it at all (the answer of one student: "I cannot be oriented in the figure, and therefore I do not know which third dimension (what a part) should be counted."). The rest of students did not solve it right, e. g. in 18 % cases there is 4.75 cm written as the result (19 cm : 4 = 4.75 cm).

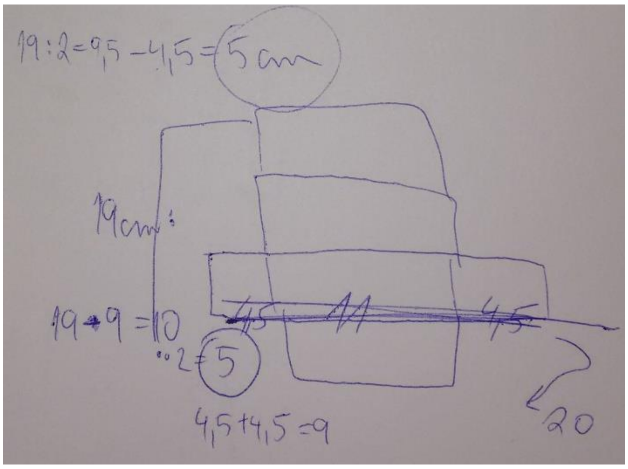

Fig. 6. The right solution of the task 09/2017.

Tasks focused on calculating the area of a planar object drawn in a square grid belong to a recurring standard task type. They occurred in 2013 (two times) and 2015, and the last time in 2016. The success of this type was acceptable, which, we assume, was the result of drilling. In 2016, however, there was a sharp decline, when only 22 % of pupils solved such a problem successfully and up to 23 % of all pupils did not solve it at all. We would like to show the assignment of this task:

2016/17 The area of each square in the grid is 25 mm² (Fig. 7). Calculate the area of the triangle DEF in cm². Express the result rounded to three decimal places.

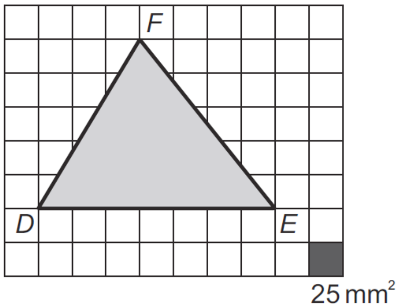

Fig. 7. Figure from the test item 2016/07.

If we make a comparison with the task of the same type (see Fig. 8), we can see that the difference in the input data lies in the dimensions of the square unit in the grid, respectively in their area, and in units in which the area is required.

In our opinion, the low success rate of this test item was due to the following reasons:

- a pupil correctly determines the length of the square side in millimetres, he/she correctly calculates the area of the triangle using the formula. But at the end a pupil lets the value of the area in square millimetres or makes a mistake in converting units of the length;

- a pupil mistakenly uses the given information – the value "25 mm²" is understood as the "25 mm", it means as the length of the square side, and farther calculates with this value.

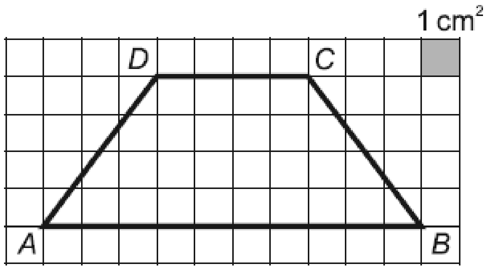

Fig. 8. The figure from the test item 2015/20.

5 The critical areas of school mathematics

The main purpose of Slovak nationwide testing T9 is to:

- compare the performance of individual pupils and schools in tests,
- obtain an image of the performance of pupils at the exit of the second level school (ISCED 2),

- monitor the level of the readiness of pupils for further study (ISCED 3),
- provide schools, decision makers and the general professional public with feedback on the level of knowledge and skills of pupils in the subjects tested, which will assist in improvement of teaching (see [8], [9], [10], [11], [12], [13]) .

The results of this testing can additionally offer another option except of the above-mentioned ones. Obtained statistical data can be more quantitative analysed in the individual testing content units or in the selected sets of test tasks. On the selected tasks it can be done deeper qualitative analyse. Finally, on the base of the analysis it can be identified strong and weak parts of school mathematics, especially the weak ones. The concept *critical areas of school mathematics* was introduced for such issue in [3] and named the areas in which students often and repeatedly fail. In this publication the school geometry has been identified as the critical area in Czech Republic and the research was based on the results of international research testing TIMSS 2007 and interviews with the teachers. A similar situation has also found in Slovakia, where the low level of results in the content unit Geometry and measurement in T9 proves the low level of competences of pupils in this field.

It is important for the teachers of all levels of mathematics education to know about the critical areas of school mathematics. In these critical areas, it would be appropriate and necessary to change teachers' approach to education. This requires cooperation with teachers from practice.

6 Conclusion

Reports to the Slovak nationwide testing of T9 as well as various studies for further testing show that the geometry is a critical area of school mathematics. Teachers often identify school geometry with geometrical constructions (see [3], [5], [6]) or well-trained procedures for solving calculations concerning the properties of planar or spatial objects. But the ability to perceive the geometry of the world is lost. In school geometry it is important to introduce modelling, discovering and real-world geometry problems. At this time, the next important area concerning school geometry is the visual representations of data, because we are surrounded by “fast” data, we need to tell as much information as possible. Images, diagrams or charts can be used for this purpose.

The role of mathematical drilling and training algorithms in a given type of task is important, but not the most important in school mathematics. The main aim of school mathematics is not to achieve good test results, but to develop mathematical thinking, not to calculate as many tasks as possible, but to understand the mathematical context in the world around us and to be able to solve different problems. The results of the test should only reflect the level of the pupil's achievement and the effectiveness of the cooperation between teacher and pupil.

In the preparation of future mathematics teacher, we suggest to:

- demonstrate the problems with the figures as specific tasks with different types of data (information),
- focus on using images in solving different types of tasks,
- devote to the creation new assignments of the problems with figures,

- create a sequence of tasks supporting elimination of the negative impact of the drill.

If we want to prepare high-quality mathematics teachers at universities, it is necessary to have information from primary and secondary schools about the issues that are problematic and properly warn future teachers of mathematics to these topics.

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