Slovak University of Technology in Bratislava
Faculty of Mechanical Engineering
17th Conference on Applied Mathematics **APLIMAT 2018**

Proceedings

AN INNOVATIVE EDUCATIONAL PATH ON MATHEMATICS AND FILM CRITICS FOR THE FLIPPED CLASSROOM

CIVERIATI Elisa (IT), TORRE Matteo (IT)

Abstract. The paper talk about an educational path that link film critics and mathematical beauty developed in a 1st class of Italian High School. The two film used to introduce the mathematical concepts are π and *Vertigo*. Some innovative aspects are contained in this educational path and the teaching method is the flipped classroom.

Keywords: scientific film, art, π, spiral

Mathematics Subject Classification: Primary 01A20, 97A40, Secondary 97A80

1 Introduction

But what ther's in a million? A lot of zeros. A nothing, a circle with a hole in the middle. This abutment, comes from the film "Sabrina" of Billy Wilder, bears witness to the way in which the film world evoked mathematics until many years ago. Yet it had greatly helped the cinema, in his birth and growth. Various mathematicians, moreover, it had loved: Gödel there went frequently, loving in particular the animated film of Walt Disney and seeing three times "Snow White". Ludwig Wittgenstein, that was not exactly a mathematician but in which philosophical thought mathematics has always been well present, adored the western and the Hollywood musicals; Norbert Wiener was so passionate about cinema to write in 1952 a letter to Alfred Hitchcock to persuade them to make a film on its Mexican adventure; Renato Caccioppoli was for many years president of the cinema club of Naples [1].

The cinema instead, until not so long ago, has ignored the aid received in its technological evolve from the mathematics and the admiration paid by many mathematicians its expressive forms. For some years, however, the situation appears to be changed radically. The interest in mathematics rose considerably. *The mathematics has become fashionable subject of theatrical performances, topic of successful film* [2]. Why this sudden interest of cinema toward the most misunderstood, and defamed discipline of human knowledge? As always happens in the explanation of a

historical and social phenomenon, it is difficult to find a single cause. Recently it has been found for example that among the public of the frequenters of exhibitions and festivals as well as between that of readers of books, the interest in mathematics has exploded with force, so that the initiatives related to the World Year of mathematics, proclaimed in 2000 by UNESCO, have often become a mass phenomenon.

Moreover the role of mathematics in contemporary society has grown immensely and the cinema, mirror more or less faithful (as all the mirrors) culture of his age, it was finally noticed that the mathematicians are directly and usefully dealing with matters of biology, economy, information technology, safety, ethics, ... and many other fields of knowledge and human making, essential to the social and economic life of the world started to become globalized.

And the didactics of mathematics is prudent of the cinema? If you exclude Emmer and few other exceptions in the Italian panorama is rare to find examples of integration of cinema in the didactics of mathematics if not in only a few cases (often interesting) wherein theater performances or short films are used for the dissemination of mathematical concepts.

In this paper we will discuss an innovative and didactic proposal, experienced in the school year 2016-2017 in a first class of Liceo Scientifico "B. Cavalieri" in Verbania, composed by 28 pupils, in the context of the educational project "Adotta science and art in your classroom" [3]. The didactic proposal foresaw the introduction and the anticipation of mathematical concepts considered complex but important for ministerial guidelines, through the use of film which functioned as "lessons" according to the spirit of the flipped classroom teaching model. The teaching methodology of the flipped classroom was chosen for the numerous possibilities that it offers to all students (in particular those LD) of approach to new topics according to the times and manner determined by the student himself, as well as to be able to capture the interest of students through means of communication that they use on a daily basis. The selected films will assume, in this experimentation, the role of "real task", a category of item being sought by the flipped didactics, not easy to define and difficult to place within the Ministerial guidelines.

2 The Flipped Classroom

Before describing the merits of the didactic proposal in question, we grant you a short detour on the meaning of Flipped Classroom. It consists in reversing the traditional didactic moments, allowing students to follow the explanations at home and performing the exercises at school, to promote active learning of the student. This choice has substantially two reasons, one social and one didactic.

In the two moments envisaged by the method Flipped Classroom both teacher and student interpret new and different roles than the current ones.The teacher has no longer necessarily the role of dispenser of knowledge, but rather should be able to direct the students on the correct road that leads them to the conquest of such knowledge. The student, on the other hand, is no longer the passive auditor of a lecture, but he/she becomes responsible for his/her own learning.

The theoretical bases of this teaching model are based on doctrines of John Dewey and Maria Montessori, beyond that in modern pedagogy that prefers the appearance workshop of learning. Dewey speaks of *active* school, that school can put the child in contact with the difficulties of the outside world, encouraging and guiding it in the strategies and actions that he is caused to perform. His school is also known as *progressive*, because the child lives and develops for degrees with the succession of experiences that lead to an education ever deeper. Dewey argued that *Education is not a preparation for life, education is life itself*. The Flipped Classroom can therefore be considered a revisitation in a modern key of Dewey's active school.

The Montessori method is mainly based on the idea that the learner must be free: free to experiment spontaneously, free to express his innate creativity, free to cultivate his own authentic interests and even free to move to reach the mastery of himself and of his own body. Also in this case the education is focused on the needs of the learner and the educator plays, just as in the flipped Classroom, a role of guidance and support.

3 The didactic path

As previously anticipated, we used the Seventh Art to speak about mathematical concepts that are present in the didactic program of the first year of high school and to introduce, in an absolutely original, the concept of beauty from the mathematical point of view. The vision of the films was assigned to the students as home work; the critical comment and analysis of mathematical aspects from this emerged were treated at school. In the following we will present a quick analysis of the two films, "π" and "Vertigo", we chose for our educational path and the motivations that led us to select them.

The first film, π (as is already apparent from the title), speaks of mathematics and science in a strict way, there are no hidden references to uncover and was chosen to introduce the concept of irrational number, both for the fascination that the number π, together with other irrational, has always aroused mathematical in evoking the concept of beauty.

In fact, exist a class of Numbers that since the Antiquity fascinate human's mind, leading them to the research of symmetries and rules even where these do not exist. Numbers represent the known and the rational for excellence. Nevertheless, from Irrational Numbers of Pythagoreans to the Golden Section, many Mathematics "mysteries" have been investigated. Several of these ideas and cultural traditions have a significant influence even on our Society, so that it is interesting to wonder which are their origins and which processes do bring their diffusion.

We have chosen to propose the integral vision of the film, but it could be appropriate to submit a shortened version, operating perhaps cuts to lighten the vision of some steps of the film and to adapt the usability at the level of the class.

The second film, "Vertigo", is decidedly more "difficult" and its analysis was possible only thanks to the skills of teachers to synthesize, in a limited time, the stylistic and cinema elements that distinguish it as a film by scientific flap. The Hitchcock's film in fact, for the complex internal architecture subtended to the whole frame (not particularly complicated to follow), lends itself well to be used as a practical "laboratory" to begin the guys at the

rudiments of the filmic analysis, introduced timidly at school by enthusiasts teachers, but that is slowly taking more and more a structured shape. The Hitchcock's masterpiece is not precisely a film on mathematics, but it approaches to scientific sphere in the moment when it transforms the golden spiral in a whole narration element, making it emblem and symbol. This bond, evocative and fascinating, was useful for students to approach with a more critical and attentive look to the numerous interdisciplinary links that exist between science and art, as well as to recognize that knowledge is not made to closed pouches, but it is necessary to seek those interconnections that make the knowledge complete and dynamic.

"12:45 Utter again my theories.
1 The nature speaks through mathematics.
2 All that surrounds us can be represented and understood through the numbers.
3 By plotting the graph of any numeric system it follows a diagram, then
everywhere in nature exist schemas".

This is the mantra that Max Choen, the protagonist of the film π, insistently repeats to spectators. Independent US movie with a very low budget and directed by Darren Aronofsky, π is presented in 1998 at Sundance Film Festival, the prestigious exhibition dedicated to the independent cinema, obtaining a good success with the critics and the public.

The stylistic figure of the film seems to be just very daring and characterizing: shot in 16 mm and with a black and white triturated that accentuates the contrasts, π catapult immediately the spectator inside its oneiric and alienating dimension and through a frenzied and anxiogenous race makes it sink in the mental universe of the protagonist.

Why π is in fact a "not history", is a journey inside and outside the mind brilliant and frustrated mathematician Max Choen, that, as a modern Icarus, moves too close to the sun - both physically, when he tells and relive the experience of him the child, when literally blinded for having wanted to look directly and too long to sunlight, both metaphorically, when he thinks he has found the essence of God translated in numerical sequence - and precipitates dramatically.

Icarus loses its wings of wax, Max loses the capacity to reduce everything to a schema to a "pure reason": can no longer dominate his mind, then decides to destroy it.
Despite the film departs from the canons of cinematography more traditional, you can find some topos of film genre.

Since the first sequences in fact we understand without a shadow of doubt that the mathematician Max Choen is a creature particular, above the lines, alienated and a little foolish (he suffers from very strong headaches that cause off stabbing pains and hallucinations); he has no significant contacts with other human beings and find security only in a reality that he can understand and control, which may thus be translated numerically.
Genius and madness in short, as in the best tradition.

But in this film this aspect is brought to exasperation: point of view is confusing, the spectator is accompanied with a swirling movement into and out of the hallucinations of Max, so increasingly excited and confused, until (filmic) reality and (oneiric) vision interweave, making the spectator a chaotic movement inside and outside the mind of Max, movement

stressed masterfully by a soundtrack hammer and visionary, consisting mainly of electronic sounds. The paranoia and the subjectivity of the protagonist are underlined by the stylistic choice of film direction shooting always over the shoulders of Max, excluding the point of view of any other character.

Obsessed with the study of π and from research of mathematical models in nature, Max seeks at all costs to find a system able to predict any event that will happen.
His Holy Grail is a number of 216 digits that could open the door to the mysteries of the Universe, a mystical number that may lead to the essence of God as the refers the Rabbi Choen (his namesake), or that could drag to collapse the global financial system.

Stops the brain Max. Entrusted to instinct, use intuition, gently says his mentor Sol, mathematician of Russian origins, also a scholar of the sequence numbers of π. Max is a Sol which is not given for winning, are black and white, just like the tiles of the table of go, game of Chinese origin playing which the two friends compete.

Life is not only made of mathematics - says Sol to his former pupil -. *I have spent more than forty years in search of diagrams decisive and I have not found anything.*

In this duality of views which do not seem to be able to meet, the game of Go becomes a symbol of that universe on which collide with the different points of view of two mathematicians: The goban (the Go game table) is a veritable micro-word; first simple and tidy, as the white stones and black are positioned arrives chaos. The possible combinations of the game are unimaginable (though not unlimited) and unpredictable, so as to be able to say that no game has ever been played twice.

For Sol a universe uncontrollable then to Max a reality that you can manipulate, win, predict.
The more the game progresses, more moves become predictable, he says in support of his theory and of course not only refers to the fact impressed on the game board. *Life is not only made of mathematics* ... or maybe yes?

"*11:15, restate my assumptions*
1. Mathematics is the language of nature
2. Everything around us can be represented and understood through numbers
3. If you graph these numbers, patterns emerge. Therefore: There are patterns everywhere in nature".

The golden spiral, Leonardo and the absolute perfection of the pythagorean *tetraktys*, the Fibonacci numbers are essential topics that are strokes in the plot of the movie (Fig. 1); And in the relationships between mathematics and the cinema, precisely these issues are of particular importance and, as has already been on several occasions written [4], have a close relationship with nature, architecture and art. Without going into too much depth in fact, they are subjects easily explained and therefore possess the great advantage of being able to be shown with effectiveness to the cinema.

And for the non-mathematical undoubtedly constitute an element of considerable charm, aspect that, united to the emotional power that the filmic image is able to elicit, instils in the spectator the desire to learn more, encouraging them to discover fascinating of perfection hidden in the world to which we belong.

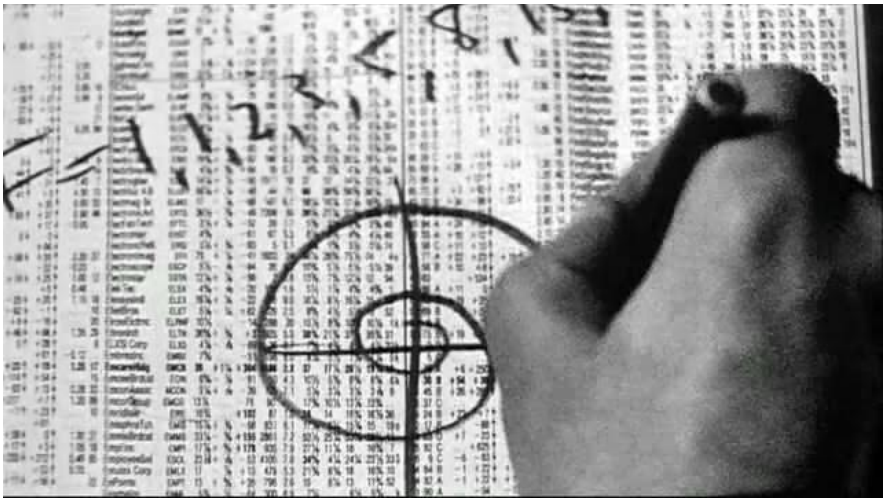

Fig. 1. Max draws a spiral with the sequence of 216 numbers of Fibonacci's Serie (frame by π movie).

In the film the trace of sublime beauty and perfection hidden in the world is represented by the shape of the spiral, a real obsession for the protagonist and in which he encounters continuously and inevitably. The shell found on the seashore, the milk that comes into contact with the surface of the coffee and draws sinuous elements, the ordered structure of an organism analyzed under a microscope that contains within itself this perfect shape, the wobbling motion of the leaves falling from the trees: the camera insists on these elements in a manner that always clearest, almost to say (to the protagonist, but also to all of us) that perfection is inherent in the nature, is beauty, is harmony, create itself and lasts forever and cannot therefore be reduced to mere rational model.

In the table of go, in the farewell of Sol at its most brilliant pupil, the tiles are arranged to form a spiral, almost to say that even in the chaos there is perfection, but is ineffable, unreachable. Only now Max includes, but it is too late. It has moved too close to the sun and not remains other than to dissolve his wings.

The quest for the perfect beauty, the desire to recreate a reality that cannot exist and the interest for the theme of the spiral unites and approaching the film π to another film light years away for epoch, writing, style, kind and budget. And fame.
It is "Vertigo", film of 1958 directed by Alfred Hitchcock (Fig. 2), now considered one of the masterpieces of master of thrill and one of the most important and significant films in the history of cinema.

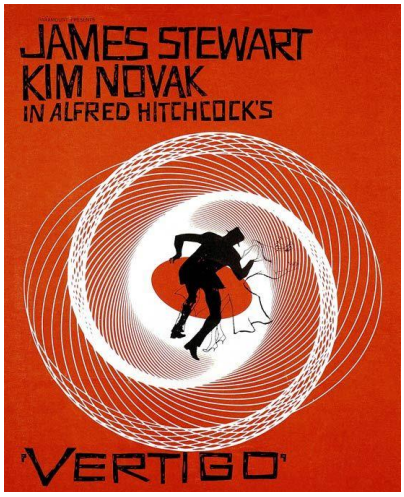

Fig. 2. "Vertigo" movie poster.

As will be known to most of us, "Vertigo" tells the story of Scottie Ferguson, a former policeman who suffers from acrophobia, who agrees to tracking the sensual and mysterious Madeleine, the wife of one of his old friend and he considered mad; the woman in fact, believing herself the reincarnation of a suicide ancestor, is thrown by a bell tower, for then reappear under other semblances. In truth - and the policeman will discover this at its own expense - everything was a deception of the friend to get rid, unpunished, the rich wife.

Exactly as happens to great literary works, much has been written about this film and much more you could write; the risk to go beyond the things that are of interest to us is very high, then we shall limit ourselves to a careful and schematic analysis, comprehensive compared to what concerns us, but not exhaustive with respect to the completeness of the cinematographic work in itself. We will limit ourselves to emphasize how this show reminders in some way the mathematical language for essentially three aspects: the iconicity, the centrality of the theme of the spiral and the scientific rigor with which each single part was connected to the other. The iconicity is given by some peculiar choices that have helped make the work a milestone in the history of cinema.

Fig. 3. the Judy's eye in the maintitles.

One of these choices is the willingness to brighten the story in San Francisco, that with its straight and boundless roads, uphill and then downhill, makes it perfectly the idea of a

movement that seems to make progress, but in reality only rotates on itself, just as the movement of the spiral, of which we will talk a little later. Iconic is the choice to make the female protagonist, Madeleine/Judy, the woman who lived twice, to quote the title with which the film was distributed in Italy, the archetype of the perfect beauty. Always remaining in this furrow, iconic is also the hairstyle of Madeleine, hair gathered with a chignon that refers to the shape of the spiral, where the gaze in subjective of protagonist lingers long.

Fig. 4. Judy/Madeleine spiral chignon.

Iconic are opening credits, that never as in this case, become an integral part of the narration and indeed provide the key to access the deep meaning of the film. Comparable to the overture of an opera, represent a succession of spiral figures which were confused between them and entrain the spectator toward that dizziness of which the film is permeated [5].

The spirals and shapes that are interwoven and rotate to the infinity, which describe a movement which can be repeated, but not completed: It is no coincidence that the spiral movement is introduced and arrested by the same shot (on the first floor of a female eye) almost as if wanting to melt together the beginning and the end.

Fig. 5. The belfry staircases where Madeleine falls, taken with a particular and innovative (at the time) technique thanks to which is simulated the movement of vertiginous fall, that emembers the continuous movement of the spiral of the maintitles (frame by "Vertigo").

Throughout the film Scottie goes in search of someone who is not there, someone who pretends to be someone else. Just as Max Choen sought perfection in numbers, the protagonist

of "Vertigo" search for perfection in feminine form of a woman but has never existed, but that he needs to recreate and pretend to be able to possess, though accepting to believe in a lie. Judy has almost completed the metamorphosis that will lead to be again Madeleine, but to the perfection of woman there is something missing: not enough clothes, hair color… Scottie needs to rediscover that perfection which has lost and asks to Judy a last act of submission; And Judy tidies up her hair in spiral chignon (Fig. 4) that distinguishes the hair of the woman loved by the protagonist, which now returns in all its evocative power.

Just as in the film previously analyzed, even here the unreachable perfection is plastically represented by a spiral. But even here the sublime beauty, or better, the perception of beauty, leads to ruin. The supreme act is an ascent that turns into ruinous fall, everything ends as is started, nothing more can be explained and all that remains is to persist in the oblivion of conscience. Just as for the explanation of complicated mathematical theorems it is essential to cut each step to have a clear vision and rigorous of everything, to understand deeply this film must disassemble and then rebuild it: only by joining together the various parts you can grasp the magnificent all of the film; for this reason we speak of scientific rigor, which is achieved with the willingness of the director to create a secret inner harmony that leads to the understanding of the theorem proposed by Hitchcock. Only a careful and trained spectator is able to grasp.

Everything becomes a circle, but the circle does not dose. Thus Erich Rohmer – the great French filmmaker - has defined "Vertigo" and perhaps really there is no most iconic way to describe what is the most mysterious, abstract and painful film of the ingenious "master of thrill".

4 Analysis of the educational impact and future perspectives

Make use of a movie, even not expressly mathematician, to introduce abstract and complex concepts is not new, but original was the choice of these two films. In the specific literature there is no trace of educational paths that integrate "Vertigo" with the introduction of analytic functions and geometry: this audacious attempt liked students involved in this experimentation, who have taken full advantage of the potential offered by the flipped methodology to deepen, also individually, the theme of beauty from the mathematical, artistic and cinema point of view, bringing in class numerous deep questions that have kindled a debate rich of content.

But we do not want to stop here. Our intention is to seek in the panorama of cinema films do not strictly mathematical, but that offer the possibility to be interpreted in a scientific key. Our idea is to be able to disseminate mathematical aspects complex even without resorting to biographical film on mathematicians, which too often put the emphasis on the eccentricity of the character and too little on scientific disclosure. Always bearing in mind that mathematics is synonymous with beauty. It is not by chance that the reason for which it is considered necessary to adopt in the architecture and in the art the fee pythagorean mathematician was born from the need to get as close as possible to the ideal of beauty, an objective which has always engaged the man who is preparing to theorising the things of the world.

Proportion, symmetry, harmony, order and beauty are the qualities to which man is naturally and that positively impact the psyche with a sense of wellness, balance and security.

The Swiss architect Max Bill (1908-1994) wrote: "*When we ask for what reason man tries to establish precise reports, rules and the exact measurements, we approach directly to the main problem […]: because man seeks the harmony? Because it goes in search of constructions based on exact measurements? Why organizes space in commensurate measures? The answer is very simple: man seeks to protect themselves against the unknown, against the uncertain*".

We want to try to transmit the same certainties of which he speaks to our students.

References

[1] EMMER M., *Visibili armonie*, Bollati Boringhieri, Torino, 2006
[2] BOTTAZZINI U., *Il flauto di Hilbert*, UTET, Torino 2003.
[3] BOCCARDI B., CAVICCHI V., FABBRI F. L., GIURGOLA G., LORENZI M.G., PAROLINI G., SARTORI A., SOLARI A., TORRE M., Adotta Scienza e Arte nella tua classe, progetto didattico-divulgativo per l'anno scolastico 2012-2013, *XCVIII Congresso Nazionale SIF*, Napoli 17-21 Settembre 2012.
[4] AMADEI D., FRANCAVIGLIA M., LORENZI M.G., TORRE M., The art of being irrational: $\sqrt{2}$ and φ in mathematics and architecture from Vitruvius to Leon Battista Alberti, in *Proceedings of 12th Conference on Applied Mathematics* APLIMAT 2013, Bratislava, Feb. 2013.
[5] TETI M., La vertigine in una spirale. La centralità della figura spiraliforme in "La donna che visse due volte", *Annali Online di Lettere*, Vol. 1, pp. 141-157, Ferrara, 2010.
[6] URL, http://www.miktex.org.
[7] URL, http://www.tug.org/texlive.
[8] URL, http://www.linkiesta.it/it/article/2015/11/21/vertigo-di-hitchcock-e-lossessione-per-la-donna-che-visse-due-volte
[9] URL, http://www.pellicolascaduta.it/wordpress/?p=834
[10] URL, http://www.900letterario.it/cinema/temi-stile-vertigo-hitchcock/
[11] URL, http://it.federicopistono.org/filmreview/recensione-pi-greco-il-teorema-del-delirio-1998
[12] EMMER, M., *Dionysus ex machina*, IV (2013), pp. 447-472
[13] VIGANO', D., *La camera oscura: il cinema tra memoria e immaginario*, vol. 1, 2002
[14] SESTI, M., *In quel film c'è un segreto: raccontare al cinema, raccontare il cinema*, Feltrinelli, Milano 2006.

Current address

Civeriati Elisa, high school teacher
I.C. "Tortona B"
Viale Kennedy 12, 15057 Tortona (AL), Italy
E-mail: elisa.cive@virgilio.it

Torre Matteo, high school teacher, free researcher
Liceo Scientifico Statale "L. B. Alberti"
Via Pontecurone 17, 15142 Valenza (AL), Italy
E-mail: matteo.torre1984@gmail.com