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VIRTUAL RECOMPOSITION AND ANALYSIS OF A SURVEYED MODEL OF THE WOODEN CRUCIFIX OF ST. PETER'S BASILICA

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Abstract. The wooden Crucifix of St. Peter's Basilica in Rome has been recently restored by a group coordinated by L. D'Alessandro and G. Capriotti and under the supervision of P. Zander for the Fabbrica di San Pietro. The work has involved a detailed survey with range based and image based methods for a three-dimensional model useful for virtual recomposition, measures, analysis and documentation. The study includes geometrical models, parametric curves and numerical algorithms.

Keywords: 3D survey, virtual recomposition, point clouds, object restoration, model analysis

Mathematics Subject Classification: Primary 65K05, 53A04; Secondary 97N50

1 Introduction

The wooden Crucifix of Saint Peter's Basilica in Rome, a marvellous artwork ascribed to the school of Pietro Cavallini in the 14th century, has been recently brought back to its original splendour by a team of restorers coordinated by Lorenza D'Alessandro and Giorgio Capriotti under the supervision of Pietro Zander for the Fabbrica di San Pietro [1]. After a temporary placement, at the end of the 2016 Jubilee of Mercy (see Fig. 1), it is now possible to admire it in the Chapel of the Blessed Sacrament (see Fig. 3a).

The restoration has involved, among other disciplines, also a detailed survey, with range based and image based methods, to construct a three-dimensional model useful for virtual recomposition, measures, mathematical analysis and documentation.

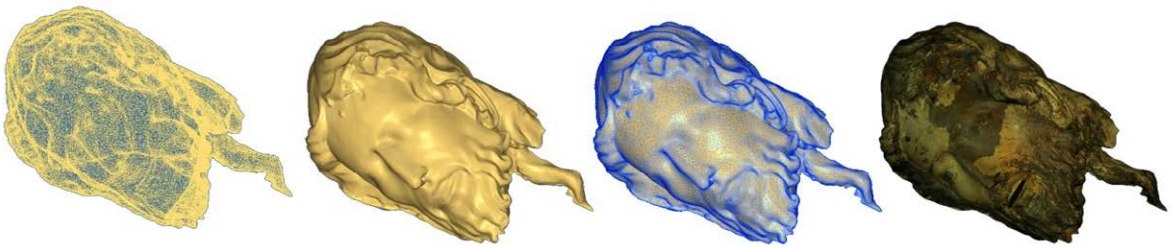
The main results of the survey procedures have been a very detailed virtual model of the Crucifix, useful for special analysis and monitoring aimed at preservation or documentation, and some specific support to restorer's study and work (as in previous collaboration using similar techniques, see [2, 3]). In particular, the visualization of different historical restoration phases (elaborated with photographic textures applied on a triangulated mesh), the virtual re-

Fig. 1. The restored Crucifix on the main altar near a column of St. Peter's Baldachin.

position of the wooden statue on the new made Cross (with data elaboration on volumes, forces, barycenter and equilibrium conditions), the left arm joint description and the wood breakings analysis to speculate on trunk diameter and drying time evolution.

2 The model

The 3D model (see Fig.2. for the detail of the head) is the result of several point cloud



(a) (b) (c) (d)

Fig. 2. Detail models of the head. (a) The connected point cloud. (b) The surface shape. (c) The triangulated surface. (d) The photographic texture mapped on the mesh.

surveys, made during the restoration process, using laser scanner and photogrammetric procedures: a laser scanner instrument returns millions of points coordinates of the object in a given reference frame whereas several pictures from different viewpoints capture its shape (with a similar point cloud) and its true colours.

(a) (b) (c)

Fig. 3. (a) The restored Crucifix placed in its actual position. (b) A smooth 3D model determined as a triangulated surface of a surveyed point cloud (c). Note the greater point density of the hand, the drapery and the blood leak.

Out of different surveys, and by an alignment of the corresponding point clouds, a single 3D model of 230.000 points has been worked out as a base for orienteering, scaling and aligning specific parts with much greater point density and accuracy: with more than 800 pictures it has been possible to produce 50 different working models and 12 final models for specific parts with a total of 10 millions points per model.

3 Data analysis

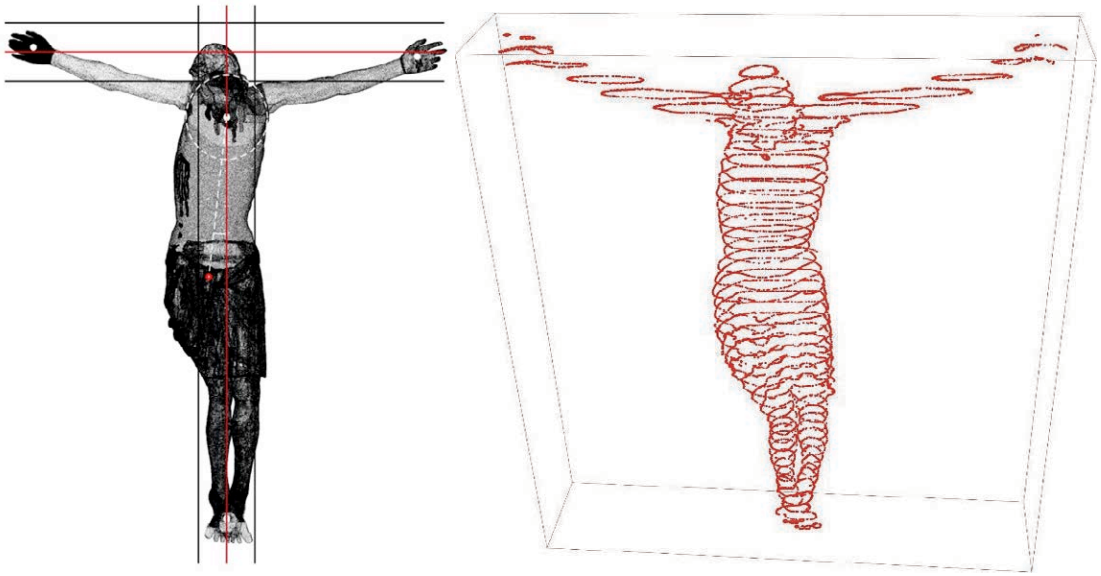
Among other elaborations, we mention here: 1) the analysis of the virtual left arm joint (see Fig.4b) together with the corresponding shoulder hole to gain some information on their relative positions and frame; 2) horizontal and vertical planar section curves [4] to check the

position of the body barycentre (see Fig. 5) and a volume estimate out of the surveyed point cloud (see Fig. 6); 3) principal wood breakings analysis to speculate on weight, trunk diameter and drying time evolution.

(a) (b)

Fig. 4. (a) Photogrammetric survey during the restoration process: image of the 3D model with true colors mapped onto the surface. (b) Triangulated surface of the left arm joint.

The position of two anchoring points (one on the back, with a metal support, taken as a pole and another on foot) identifies a vertical axis: the calculated barycentre is sloping 7° with respect to the pole (see Fig. 5a).
The back anchoring point has the same height of the centre of mass of the upper part of the body, on the vertical axis, at a distance of 8 cm from the Cross surface: this datum gives a



(a) (b)

Fig. 5. (a) Position on the Cross: the barycentre (red dot) is slightly sloping with respect to the vertical axis through the anchoring points (white dots). (b) Horizontal section curves.

measured explanation of the position of the security support.

Given the dimensions of the Crucifix (height 214 cm, wide 197 cm), the estimated volume of the wooden sculpture has been calculated from cylindrical slices of horizontal section curves (see Fig. 6): their number has been increased for high accuracy to give a volume of 137 dm^3. The surface area has been evaluated directly on the triangulated model in 288 dm^2.

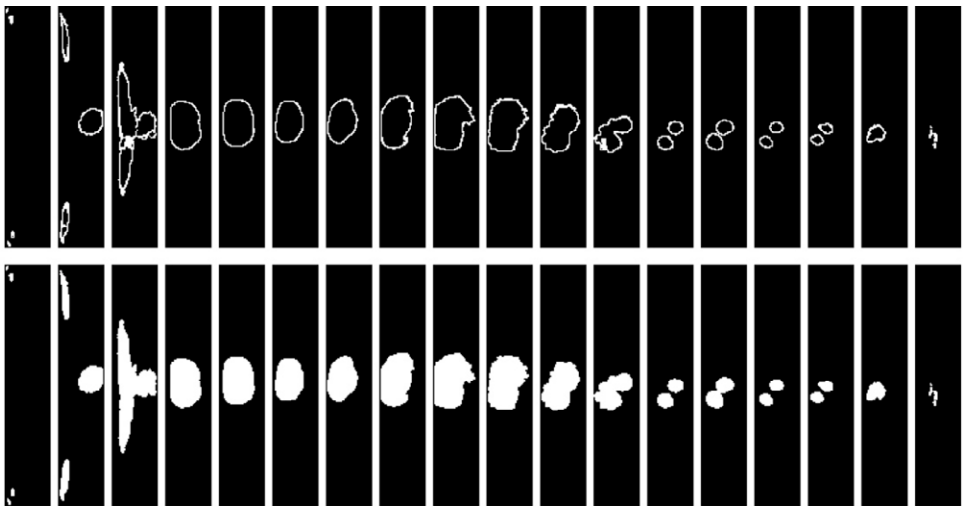
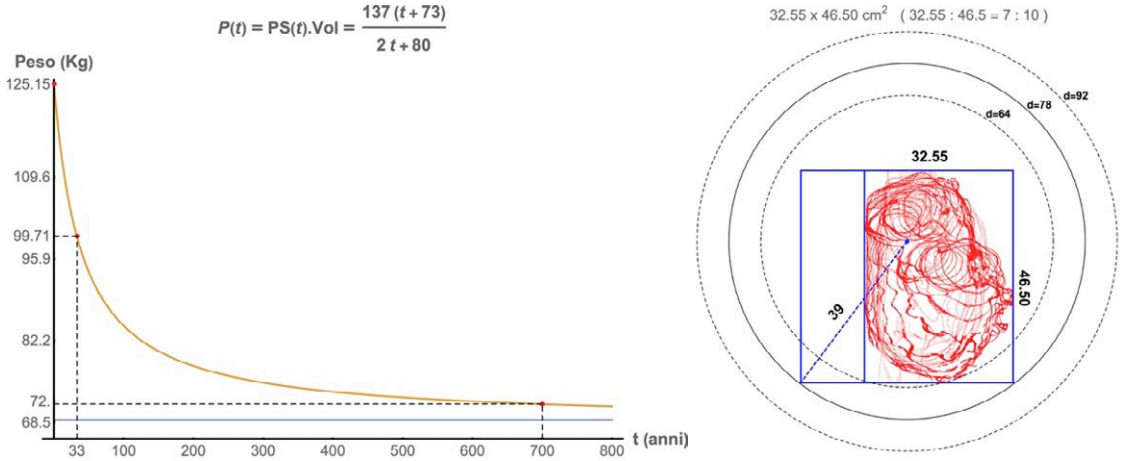


Fig. 6. Horizontal sections and the corresponding cylindrical slices to get a volume estimate out of the point cloud model.

The weight evolution curve (see Fig. 7a) is a rational interpolation of the known data of the new made Cross (of the same walnut wood of the Crucifix, dryed for 33 years) and data elaborated from the virtual model.



(a) (b)

Fig. 7. (a) Drying time evolution of the wooden Crucifix. The known data are the three red dots. (b) Walnut trunk diameter hypothesis given the central position of the marrow and the red sections of Fig.5b.

With the simplifying hypothesis of a rational function of type f(t) = (a t+b)/(c t+d) one can fix the parameters using the calculated wooden sculpture volume (137 dm^3) and an estimate on its actual weight (72 kg), the volume and weight of the new Cross (elaborated from its surveyed shape: 82.44 dm^3 and 60 kg) together with its measured drying time and the standard intervals for specific weight of walnut green (0.91-0.92) and dryed (0.44-0.68) wood.
The variation inside the two intervals depends on wood density and therefore is increasing going from center to the trunk surface (see Fig. 7b).

The simplified model of the weight evolution as a function of time (in years) can be given as the simple expression f(t) = 137 (t+73)/(2t+80).

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