Abstract. The design of tall architectures building assumed for modern architects a particular compositive challenges: not only following the new technical potentiality derived from innovative materials for construction (i.e. iron, steel, reinforced concrete etc.), but also evocating different essential symbols through monolithic shapes. Some recent building sites in Milan offer the opportunity to focus the attention on some evidences in the evolution of design techniques toward the composition of these buildings, in comparison between the XX century analogue orthogonal drafting machine and the actual complex digital 3D (4D) tools. These two drawing techniques could be considered as the output devices of different algorithms approaching the same gravity challenge to which any architecture design must afford. The verticality of the building structures allows the overlapping of the horizontal floors following the static sciences: the construction and available materials technique could obtain different solutions essentially in reason of calculation process and aesthetical approaches defined by design concept and drawing procedures.

Keywords: mathematics and visual arts, mathematics modelling, application of mathematics, music, language, architecture computer science applications, mathematics and architecture

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1 Milan skyline

The Milanese urban landscape has profoundly changed in these last decades. Considering the most significant episodes of post-war architectural construction, and in particular the most characteristic building typology of modernity - particularly skyscrapers or tall buildings - we can see how, between the generation of twentieth century buildings and those of the last decade, is recognizable a substantial diversity. [1]
Fig. 1. Milan skyline with modern and historical building (in first evidence the Torre Velasca and Duomo) and a sequence of contemporary new skyscrapers (from left to right: Unicredit tower; Solaria tower; Regione Lombardia tower; Diamante tower; Torre Galfa; torre piazza Repubblica e Grattacielo Pirelli) (photography of architecture by the author).

Fig. 2. a, b, c, d. Large Milano Town model (skyline view), and a sequence of morphological models of urban tall building in Milan after the post-2nd war urban development: actually exibited at Urban Centre of di Milano in piazza Scala, previously realized for the Biennale di Venezia (editor Arch. Cino Zucchi) (photography of architecture by the author, with the courtesy of: Urban Center).
2 Analogical composition and digital morphogenesis

It is not only a question of style or construction procedures or building materials that have radically changed by a few decades of difference, although many techniques have highly evolved and refined. I will try to demonstrate and recognize how this evident design modification, above all is due to the innovation of the design and calculation methods of the architectural project [2]. In fact the first generations of skyscrapers were designed and calculated according to analogical procedures, with the mathematical support of structuralists and experts of static load, with three-dimensional physical models in scale for the static testing of resistances exceeding the pure weight of the building itself (like the wind, vibrations or other stressing factors) [3].

The analogic composition - as a stabilized and harmonious equilibrium between the parts- have been progressively approached, matched and replaced by the digital elaboration of morphologies - produced according to the multiple parameters of algorithms - selecting the choice as the best solution. A symbolic representation of the former analogic algorithm and desktop, in its procedure and devices (see: Fig 3 and quoted in §3, fig 5 and 6.). The former hypothesis evaluation procedure (heuristic sketches and formal intuition), typical of the analogical project, has progressively moved to digital designed parameters that are analytically evaluated and processed, up to the massive production of results from which the designer "selects" the one in which it recognizes as the most suitable for his concept.

Fig. 3 a, b. The mechanical joint with a protractor's goniometer knob, allows the draftsman to maintain the orthogonal Cartesian coordinates on the whole plane of the drawing template, possibly varying the coherent inclinations. While the left hand of the draftsman guides the device to convey the accuracy of the analog algorithm to the field of the draw on which to operate, the right hand will perform the calibrated traces by the two right-angle millimeter scales.

An intermediate generation of projects (years 1990 - 2000) has acquired the calculation procedures according to the first generation of digital softwares, which have effectively allowed to accelerate the phases of design and calculation, being able to interlace directly the drawn shapes and the numerical values as parameters that could be examined by the designer into the same computational process. First of all, this allowed to verify immediately - correct, modify and validate- the various phases of the original project - bidimensionally in plan and sections - and following in a digitalized process that, setting the drawing as templates of 3D matrix virtual spatialized data, make able to modify the three-dimensional view -being a virtual model- in a visually effective way, essentially due to an automated generated geometric perspective, on the monitor display or by a printer.
Fig. 4 a, b, c, d. Some examples of educational projects of small architectures composed and rendered according to a first choice of material surfaces, simulated in perspective view and chiaroscuro the environment in which they are inserted. Softwares: Sketchup and Kubity for VR with QRC interactive database (didactic design exercises at the Brera Art School, Laboratory of Design: Prof. Federico Brunetti, with the collaboration of Arch. Marco Valentino A.S.: 2016-2017).

In this first phase, the digitalization of the project was soon appreciated both for the dematerialization of the design process that transformed drawings on paper into digital elaborated files obtaining their effective transmissibility and sharing. Besides also for the easy visual illumination rendering and surface texture simulations that progressively made the design presentation simulated as useful photorealistic images substitute for the evaluation of the project.

Subsequently - over the course of a few decades and with considerable speed - the digital relationship between the different design skills (architects – static engineers - plant engineering, energy engineering, aerodynamic etc.) were able to integrate with the relative complexity of specific approach and features, thanks to new softwares that were able to align different kind of drawing files into complex layered systems. These new generation of softwares -often originally conceived into high-tech laboratories in function of more advanced technologies such as aerospace- become a shareable project tool.

Also for this reason, today we can see futuristic and eterogeneous projects and realizations, rather detached from the usual image and usual morphology of the urban fabric: each building demonstrates not only the design concept of the architect and the team that conceived it, but also the computational innovations that made them possible drawing and calculation of shapes. We face with the apparent paradox of highly technological design solutions that assume assonances of biological or natural morphologies, since managed by digital algorithm able to propose dynamic solutions to an univocal original problem.

It should be noted that such process of integrated evolved design, is finalized to compare the competences (and the various specific softwares) in order to forecast accurately not only the dynamic structural stability, but also the thermodynamic behavior of the building as a whole environmental impact (defined “Multiphysics”), its potential and planned and effective maintenance procedures (BIM), and the visual and potentially ecosystemic relations of the building with in its environment. It should also be remembered that the digital technologies of VR / AR (stereo interactive virtual or augmented reality) have a remarkable application precisely for the perceptual, interactive and immersive simulation into virtual space and environmental qualities for the foreshadowed fruition of design.
solutions and decisions [4]. Related to this same devices, some new paradigm of visual experience of the space has been also introduced by digital spherical cameras –and the sw needed for the image matching- that allow to perceive and broadcast a full 360° view of the environment, even in remote without the presence of the shooting operator [5].

3 Analogic drawing desktop to Digital drawing desktop

Merely as a symbolic representation of the changing occurred from Analogic drawing desktop to Digital drawing desktop, are following presented two allegoric make up of scenes representing the methodologies in these two era, so near to us, but deeply evolved.

In the first are in evidenced the typical books of the modern architect education and debate, referring the proportional modularization into the building composition (Fig: 5). The modern architectural movement sustained the actuality and legitimation of its methodology as possible application concerning the standardization format for industrial production of objects and architectures. Avoiding to recognize any explicit validity about traditional architecture proportioning methods (inspired by to archetipical principles of utilitas, firmitas venistas – i.e. functional efficiency, structural stability, symbolic beauty-) Le Corbusier in his famous treatise - manifesto "Le Modulor" recognizes the golden sections as a compositional methodology conceived of universal value for modern architecture and rediscovered the theoretical and practical significance of proportions. This approach relegated to archaeological researches the simple recognition of their traditional value in historical monuments [6]. In fact the modern design drawing practice, avoiding to be based on the imitation of recurrent stylistic models – considered as endless arguments of academic discussions- rediscovered the power of analogical algorithm, as golden section or even graphic computation, to organize efficient composition of plans, sections and volumes. The structural calibration of reinforced concrete, or steel, constructions were definitely delegated to static engineering competence, and the architectural composite research believed to be free to be concerned on harmonic and socially useful research [7]. But this creative and productive approach has also some strong ideological and self - referenced features, in fact any effective debate for the unification of modern Arts around golden proportion in mathematics had some explicit failure [8]. For that generation of architects the icon of the New York skyline was surely a strong challenge in comparison with Italian post war architects possibilities, but the wise use of well balanced harmonies, by means of the analogic algorithm of orthogonal drawing, aided by the proportional computation by slide rule and the use of compass, were able to realize emblematic solution, still admired after more than half a century. Some other typical analogic devices, as a typewriter machine ( a real digital device !), a photographic telemeter camera and a stereoscopic slides viewer, were other usual tools of that ingenius age [9] [10] [11].

On the digital drawing desktop environment some new procedure has been taking place (Fig: 6). The shape of objects can be obtained as the result of optimized data of efficiency from the forecast of dynamic phenomena. The horizontal plans and the vertical development of a building can be modified progressively obtaining the best possible results matching static resilience with symbolic evolutive design, environmental energy consumption. The emneneutical inspiration sources are more rarely books, but huge databases of virtual models catalogues reach the designer desktop everywhere a good web connection can arrive. The support of the drawing, since now as essential documentation material, in no more a sheet of paper, but a cooled digital stocking memory, where the virtual models of tridimensional building are computed, stored and renderizable in any interactive visualization. The pencil or the compass tracks are potentially substituted by the hand gestures of the designer by a mouse or a drawing tablet, and the drawing attitude of the designer is traced by a scanner pen. The sharpness of the drawing is substituted by the precision of the pixels, pure positional numbers, and being comparable to a laser light ray able to scan or measure any close or remote detail. The proportionate experience of the sight on the environment is augmented
by full field *spherical cameras* and *stereoscopic smartglasses devices*, whose overlapping vision can be activated by *QRC* targets [12]. The computation performances of digital algorithm can expand our knowledge toward infinitesimal and complex shapes preview, may be potentially closer than ever before, very similar to the naturalistic biological evolving configurations—as fractals– where matter, energy time and space are inextricably intwined.

4  **A XX th century emblematic modern architecture: The Galfa tower**

The first case study here presented is a building realized after the second World War economic development, when tall buildings and towers represented the first new skyline of the directional business center near the Central Railways station. The Torre Galfa (1956-1959) is an emblematic example of modern building -currently undergoing a "revitalization" by the actual Real Estate property Unipol Insurance company, following the project of studio BG&K associati - in the area near the the famous Pirelli skyscraper designed by Gio Ponti. In this work Arch. Melchiorre Bega (1898-1976) designed it as an elegant example of a particularly proportioned and refined composition in its prismatic essentiality in the context of a lively debate, and other coeval examples of the Milanese modern architectural movement,. Originally designed and calculated in the era of the analogical project (with Papini (1924-71), P.A. Rognoni 1921), ing. Luigi Antonietti (1924-2010), Arturo Danusso (1880-1968)), the building is articulated by a hypogeum plate with a wide structural grid, extended on the entire available urban block, from which it rises with a relation to the lower floors of a body made by low and long-range transversal factory of services. The construction of the tower - initially planned for a 29-story hypothesis high 99.44 m, results in the final version 102.37 m. and 30 floors. The essential and almost hermetic prismatic shape has a plant of 37.50 x 15.75 m., compose the floor with 21 modules in depth and 50 length by a square grid of 75 cm. from the side. The supporting structure of the floors is not resolved in pillars, but with vertical septa (75 cm thick at the base of the building) that support it with a variable pitch between 9.70 and 3.90m. From the 24th, these sects diverge into two thinner pillars. Just as the plant and the section show a decidedly accurate modular composition and numerology (of which in the sketches that we present only some initial interpretative hypotheses), the original façade was designed with a planar articulation by a series of aluminum modules with windows and vertical / horizontal tilting windows, designed with an exemplary rhythmic sequence both in the single module and in the overall surface area. The morphological characteristic of the building in its prismatic essentiality lies both in the planimetric proportions on the façades, and in the whole volume, expresses an unexceptionable order. The compositional rules of this way of planning are rooted in the classicism of golden sections, which now we can only deduce some interpretative hypotheses, while the studies on the geometric articulation of proportions are normally not published and implicit in the final executive project. The analogical algorithms were not less precise in the graphic composition of golden proportions, basing their mathematics on the use of the compass line and circle to define the rhythm of the partitions, although they remained in field of mutual relations and three-dimensional orthogonality, indeed finding a special form of modern abstract beauty and visual harmony in this complex simplicity.
The current state of revitalization of this building has allowed us to observe and document it both before the works, followed after a long period of decommissioning [15], and during the phases of its skeletal transparency, when the previous facades, currently technically obsolete, were replaced by new ones cells in glass doors and windows that - together with other technological, typological and functional upgrades- bring to actual thermo-acoustic insulation standards requirements. In this building we can observe the continuity of some fundamental concepts. The ancient "canon" of the Greek
classicism in which *téchne* (*i.e.*: art/technique) were merged (in heritage of thousand-year tradition of the golden proportion analogic algorithm that connect in the drawing activity the human intelligence with the re-creation in the project (nature - morphology / geometry - beauty), and the modern relationship built on the rationality between order and form, derived from LeCorbusier’s Modulor where efficiency and anthropocentrism tried to establish a new form of equilibrium (ergonomic standard - reproduction / production) [16].

Fig. 7 a, b, c, d. Galfa Tower, 2015-2017: phases of revitalizing of the building, works in progress of refurbishing; façade details, models (Urban Center and study maquette).
Fig. 8 a, b, c. Galfa Tower: Authors’ heuristic hand drawn proportional sketches, with hypothetical first interpretation, deduced from edited original drawing of GALFA tower. (some inevitable approximations are certainly intrinsic in these sketches, due also to the fact that they are not based on original copies of the project drawing, but simply derived from reproductions edited in recent publications; other further studies are in progress).

Fig. 9 a, b, c. Hadid Tower: Authors’ heuristic proportional didactical sketches, hand drawn on blackboard, following Zaha Hadid studio presentation, as seen at Generali Hadid Tower during the press event book presentation on 17th.10.2017.
5 Hadid Generali Tower

The second case study belong to the last generation of skyscrapers designed for the town. The site where this urban development take place was the former Fiera di Milano location, previously here located since quite a century ago, and now moved to a more infrastructurated areas in external part of Milan [17]. The project, contextual the disposition of the former setting towards the proposal for the new urban design, had been the occasion for an international architectural challenge where the most important “Archistar” were requested to work, in collaboration with young Italian architectural ateliers and urban national developer, to propose their solution. This tower is a part of a complex setting defined by the winner for the development challenge: the Citylife group. The tower is settled in the visual landmark long distance perspective fulcrum, where the actual urban development introduces a three towers system, visually strictly related to the near connection to the highway approach to the town, that assumes the role of new symbols in the iconic architectural constellation in Milan metropolitan area. The first two skyscrapers: Zaha Hadid tower and Arata Isozaki towers have been already finished, the third Daniel Libeskind tower is going to be in starting phase [18] [19] [20].

Fig. 10 (a). (left): The original district map: a square area settled with diagonals organizes the subdivision of the space. A XIX century isometric military excercitations field became an ideal exhibitions platform location, with simulated urban roads partition and large covered spaces designed to host the Fair events. This “Fair machine” worked efficently here since 1926, being already based in Milan a former great background: the Universal Exposition in 1906.

Fig. 10 (b). (Right): the actual urban development plan, as presented in an orientation panel outdoor the public park entrance, where the three towers, and block of residences at their neighbors, have been built.

The Citylife towers urban development is located in a strong infrastructural, surface and underground, lines area. The social impact in this part of the town, when all the functions will be operative, will be quite important. Some block of residential buildings, and a large green area and a market district have been previously realized. The skyscraper here in evidence has been conceived and realized by the Zaha Hadid studio and it is characterized by a clear and impressive feature in the vertical torsion that appear in its vertical
façade and that needed important structural research in the design of pillar systems, disposed in a proportionated inclination, following the general shape.

This master design concept concerned the structural engineer to elaborate a specific algorithm to generate a complex and dynamic virtual model to forecast and balance any consequence of this choice. The double symmetric torsion of this vertical grid of pillars, also sustained by an inner structural core tower where all vertical direct communications are concentrated, results so particularly strong and elegant even if morphologically unique in the town landscape. The digital procedure here adopted allowed to realize, and verify by laser scanner survey and drone inspections, all the phases of construction.

Finally the façade glass surface has been realized following the general mathematic curve of each windows position. Hadid architect studio, REDESCO engineering and CMB construction company, during the inauguration of the building, defined this design algorithm as a 5D procedure, since includes the 3D basic dimensions, but even the timing and building sequence aspects, and all related documentation attaches and BIM model and information for maintenance.

Fig. 11 a, b, c, d. Hadid Tower at Citylife, Milano 2017: final view in urban skyline and façade detail. Some figures of this skyscraper: Height: Architectural: 177.4 m / 582 ft; Floors Above Ground 44; 3,200 people workplaces; Parking Spaces 300; Owner Generali Group; Design Zaha Hadid Architects; Structural Engineer Redesco Progetti srl, MEP Engineer , Max Fordham LLP; Manens-Tifs; Façade:Arup; LEED
Fig. 12. Hadid Tower at Citylife, Milano 2017-2014: final view in urban skyline, and former phase of the construction of (photography of architecture by Federico Brunetti)
6.1 First Conclusion. Some methodological relations about proportions and morphology between analogical and digital design procedure.

The verticality of the building structures allows the overlapping of the horizontal floors following the static sciences: the construction and available materials technique could obtain different solutions essentially in reason of calculation process and make possible options in aesthetic approach by innovative design concept and drawing procedures.

As I could examine through the selected case study, the elaboration of the design procedure of an architectonic artefact has deeply modified in the last half century. The example of some tall building construction, in reason of its extreme challenge toward gravity forces, exemplify how the change in the approach and innovation of different drawing algorithm, from analogic to digital, driving to diverging result. The same project procedure has deeply modified each of its steps: from the technology of survey in the environment for the capture of measures, data, images, that now allows a full analytic experience of the site [21], to the changes in the design concept that, thanks to new impredicatable power in computational process that make possible to dematerialize drawing procedures and invent unpredictable aestetical approaches, also by a broader source of information as ermeneutics of the project.

The result of design can be immediately verified not only on flat technical drawing, certainly useful for the building site executive realization, but directly in comparison into virtual modelling. This fact makes possible any check about interdisciplinary technique interference before and during the construction phases and to preview for maintenance and energetic behaviour of the artefact in the environment. The model itself, for all the defined feature that are located in its virtual structure, can be considered as a shared benchmark for any control in execution phases.

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\text{design concept} \rightarrow \text{computational process} \downarrow \text{Modelling (analogic – virtual – BIM )}
\]

\[
\text{case study - observation} \xRightarrow{\text{survey}} \text{drawing algorithm} \xRightarrow{\text{architectural project - executive realization}} \text{effective control test – function & maintenance}
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\text{Drawing procedures} \rightarrow \text{aestetical approaches.}
\]

\[
\text{ermeneutics}
\]

As many other masterful scholars already put in evidence [22], it is quite clear how much the digital innovation can optimized the design procedure, but it is also evident that the actual challenge and important opportunity to develop new paradigms in proportion and composition of architecture, that can be more easily finalized to the human requirements into the complex balance with the natural world. The conformative effects in progress for the new mathematics of drawing algorithm [23] and the new responsibility for designer creativity and social effect of architecture is in evident development.
6.2 Open conclusion. The “3 worlds”; town and the value of interpersonal communication.

The experience of practice and theory about design drawing algorithm is deeply related to the problems of our age, since every sketches that is drawn can potentially become a sign for a future project. It becomes more evident what the great philosopher of Science and Communication Karl Popper pointed out some decades ago: the World 3 (theoretical concepts or virtual environment) can clearly effect retroactive consequence on the World 2 (sensorial subjective experience), and cause following human consequence to the care of World 1 (raw material reality: i.e.: Nature) [24].

Far away to be considered these as mere theoretical considerations like here to quote the conclusion remarked at the 2017 International Tall Buildings Conference held in Milano, by Steve Watts, CTBUH Treasurer Steve Watts, Partner, Alinea Consulting. He closed the conference and shared his thoughts on the modern cyber-psychologist: “...People concerned about smart technologies and the intensive use of internet and social media, who could, as a result, be destroying social, physical relationships. Eighty-three percent of young people in London succumb to loneliness, and that is one reason why developers there tend to invest heavily in common areas rather than private ones. There is a vision of encouraging social intercourse. The event closed with a final point of emphasis: skyscraper development is related to technology, but also to other aspects that create additional profit avenues. Despite this, even the biggest projects can fail: not because of lack of technology, but because of failures in interpersonal communication.” [25].

We must consider and appreciate, in our specific case of study, how much the surrounding environment of the town has received, during the years, a good impact of these tall buildings. In particular for Citylife three towers development, where actually a residential buildings complex around the towers, stylish commercial center and a very large green area surround the skyscrapers. Anyway an accurate planning of well combined mix of urban functions must be organized with this kind of tall building, in relation to the very strong social impact that they can introduce into the metropolitan style of life. The widespread success of skyscrapers in urban new development, overall in new developing countries, must be followed through an adequate consideration of their symbolic power, and with the deep meaning of humans as citizens. Technology innovations and anthropological philosophy can here find new, or old, matter of dialogue [26] [27]

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