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THE ABILITY OF PRIMARY SCHOOL PUPILS TO SOLVE COMBINATORIAL TASKS

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Abstract. The paper describes the results of a survey which investigated the success rate of primary school pupils in solving basic types of combinatorial tasks that can be used at elementary school and the methods used to solve them. A part of the paper also includes the information on the result of a questionnaire survey focused on primary school teachers in terms of their approach to including and solving combinatorial tasks in teaching mathematics.

Keywords: ability to solve problems, combinatorial problems, primary education

Mathematics Subject Classification: Primary 97-02

1 Introduction

Combinatorics is one of the oldest branches of discrete mathematics, dating back to the 16th century when games of chance played a key role in society life (Abramovich & Pieper, 1996). Combinatorics is a significant component of the mathematics curriculum, comprising a rich structure of powerful principles that underline several other areas such as counting, computation, and probability (Borovcnik, Peard 1996). Solving of combinatorial problems helps pupils to develop the use of heuristic strategies for solving problems. Applying these unconventional ways of solving problems also improves the communication among pupils. (Doulík & all 2016) Despite its importance in the mathematics curriculum, combinatorics continues to remain neglected, particularly at the elementary school level (English 2005).

The paper is one of the outputs of the scientific project SGS 21162 "Combinatorics at Elementary School - Activating Activities for the Development of Logical and Combinatorial Reasoning of Students". The research that is being carried out as part of the project's solution is focused on generating activating activities that will help develop pupils logical and combinatorial thinking. As it turns out, solving combinatorial tasks seems rather difficult for many pupils. Therefore, one of the objectives of the project is to verify these activities in the teaching of mathematics at primary school level. Another aim of this project is to use the results to improve the educational process of future elementary school teachers at the Faculty of Science, Humanities and Education at the Technical University of Liberec. We have been monitoring not only the pupils success in solving combinatorial tasks but also the selected solving strategies.

In order to gain a more comprehensive overview of the whole situation, the survey also included a questionnaire survey focused on primary school teachers. Its goal was to map the approach of elementary school teachers to including and solving combinatorial problems in teaching mathematics.

2 Theoretical background

Combinatorics is an essential component of discrete mathematics and, as such, it has an important role to play in school mathematics. In 1970, Kapur presented the following reasons, which could still be valid, to justify the teaching of elementary Combinatorics at school:

- Since it does not depend on calculus, it has suitable problems for different grades; usually very challenging problems can be discussed with pupils, so that they discover the need for more mathematics to be created;
- It can be used to train pupils in enumeration, making conjectures, generalization and systematic thinking; it can help the development of many concepts, such as equivalence and order relations, function, sample, etc.
- Many applications in different fields can be presented.

Therefore, our research focuses on the development of pupils´ combinatorial reasoning at primary school level. School combinatorics is an essential part of the mathematical culture of education. Many combinatorial problems can be formulated very easily, however, their solution is often very difficult. The importance of children's combinatorial reasoning in analyzing sample space has been evident in several studies (e.g. Benson & Jones 1999; Johnson, Jones, Thornton, Langrall, & Rous 1998; Nisbet et al., 2000; Zimmermann & Jones, 2002). The results of these studies show the difficulties faced by pupils in solving problems that require pupils to employ combinatorial reasoning.

In order to focus on developing pupils competencies in the area of solving combinatorial tasks, it was first necessary to map the current state which included not only the pupils ability to solve combinatorial tasks but also finding out what strategies they use most often to solve them. Various studies (e.g. Inhelder & Piaget 1975), which used "colored counter" tasks, dealt with the verification of pupils ability to solve combinatorial problems.

The colored counters task was part of an investigation into children's development of the idea of chance. In another task, children were presented with sets of counters: each set was a different color, and the children were asked to create as many different pairs of counters as possible.

The pupils performance in experiment suggested that pupils generate combinations only in an empirical manner by randomly associating two elements at a time (i.e., there is a lack of systematic method).

In one such study (English, 1991), 50 children aged between 4.5 years and 9.8 years were individually administered a series of 7 novel tasks that involved the dressing of cardboard toy bears (placed on stands) in all possible different outfits. The results of the study revealed a series of solution paths used by the children in solving the set of problem tasks. These paths ranged from random item selection through to a systematic pattern in item choice, reflecting increasing sophistication in solution procedure.

A combinatorial task/problem is a problem that leads towards the creation of various configurations and diagrams (Fuchs 2000). We can create configurations and diagrams by e.g. selecting elements from a predetermined finite set. In that case, it is important to distinguish whether the order of the selections matters or not. However, it does not necessarily have to be a selection of elements we are talking about. A combinatorial task might also involve a reorganisation of the set, change of its properties, etc. Such combinatorial problems also facilitate the development of enumeration processes, as well as conjectures, generalisations, and systematic thinking (English 2005).

3 The questionnaire survey

The questionnaire survey focused on primary school teachers in terms of their approach to including and solving combinatorial tasks in teaching mathematics. A total of 52 teachers, teaching at primary school level, participated in the survey. Their teaching experience ranged from 2 to 35 years. 7 elementary schools in the Liberec region joined the survey. The questionnaire was anonymous. It comprised a total of 6 questions, including 3 with closed and 3 with open answers (Table 1).

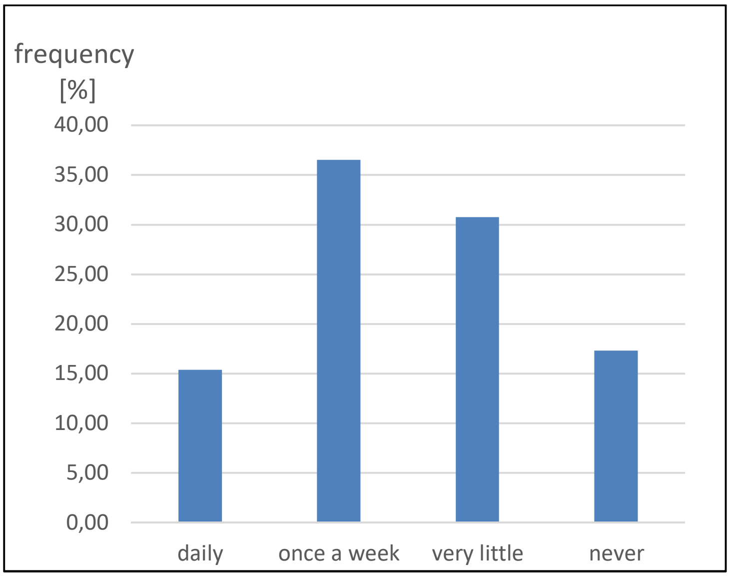
<table>
<tr><td colspan="7">Sex:</td></tr>
<tr><td colspan="7">Taught Year:</td></tr>
<tr><td colspan="7">Teaching Practice Length:</td></tr>
<tr><td rowspan="3">1</td><td colspan="6">What mathematics textbooks (publishing house) do you use in teaching - mark with a cross</td></tr>
<tr><td>Prodos</td><td>Alter</td><td>SPN</td><td>Fortuna</td><td>Studio 1+1</td><td>Other</td></tr>
<tr><td colspan="6">Specify other used textbooks:</td></tr>
<tr><td rowspan="2">2</td><td colspan="6">What types of task do you assign most often - select the appropriate option, mark with a cross</td></tr>
<tr><td>Counting (arithmetic)</td><td>Word problem</td><td>Geometric</td><td colspan="3">Other</td></tr>
</table>

	Specify other assigned tasks:					
3	**Do you think that textbooks include enough combinatorial tasks? Select the appropriate option, mark with a cross.**					
	Yes		**No**		**Unable to say**	
4	**Specify what types of combinatorial tasks you solve most often with your pupils:**					
5	**What manner (method) do you employ to solve combinatorial tasks?**					
6	**How often do you solve combinatorial tasks with your pupils?**					

Tab. 1. Questionnaire for teachers.

When assessing the data, we primarily focused on the following two questions:

a) "How often do you solve combinatorial tasks with pupils?"
From the available data it is clear that the frequency of combinatorial tasks was split up into four groups: daily, once a week, seldom and never. Graph 1 shows the percentage of individual groups.

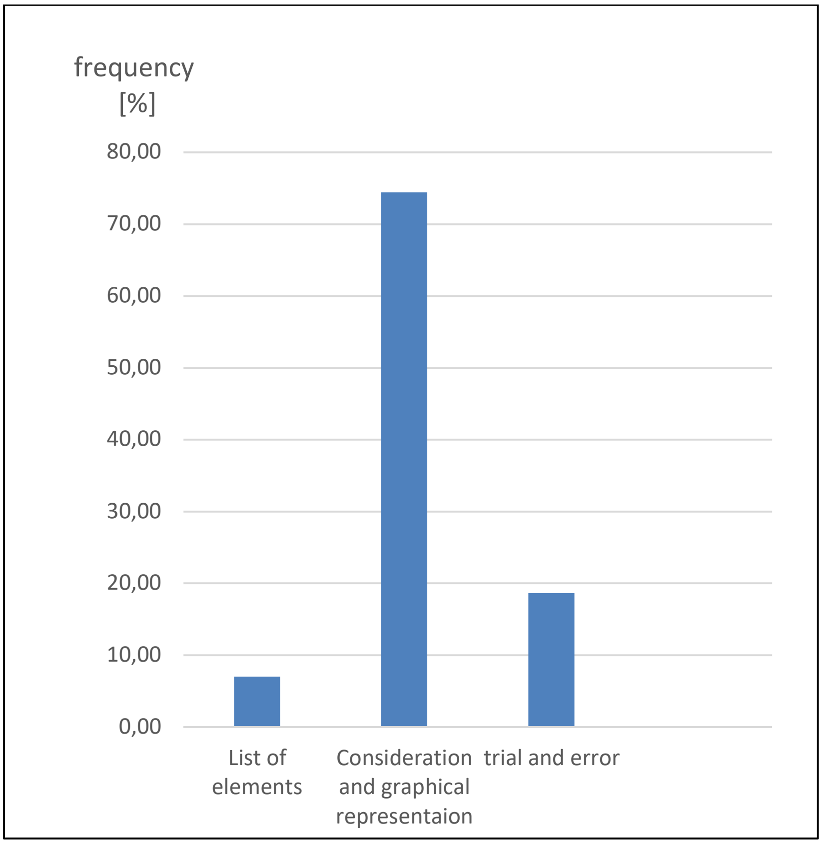


Graph 1. Percentage frequency of use of combinatorial tasks.

It is clear from Graph 1 that 48% of the participating teachers seldom or never assign combinatorial tasks to pupils. This is partly due to the kind of textbooks, used by teachers. The analysis of textbooks (Břehovský, Příhonská 2017) shows that the average percentage representation of combinatorial tasks in the textbooks is 6.3%. The average total number of all tasks in a textbook is 216, meaning the average number of combinatorial tasks in each textbook is 13. The questionnaire analysis shows a certain correlation between the combinatorial assignment frequency and the textbooks used.

b) "What manner (method) do you use to solve combinatorial tasks?"

It is clear from the questionnaire evaluation that teachers use three methods to solve combinatorial tasks: list of elements, consideration and graphical representation, and trial and error strategy. Graph 2 shows the percentage frequency of the individual solutions used. It is evident that in more than 70% of cases, teachers use the *consideration and graphical representation* strategy to solve combinatorial tasks.



Graph 2. Percentage frequency of combinatorial tasks solution strategy.

The available data did not enable a clear identification of the type of combinatorial tasks that teachers solve with the pupils most often. The survey results suggest that the selection of tasks depends primarily on the type of textbook used. The analysis of textbooks (Břehovský, Příhonská 2017) shows that diversity of combinatorial tasks types in mathematics textbooks for the primary school level is not large.

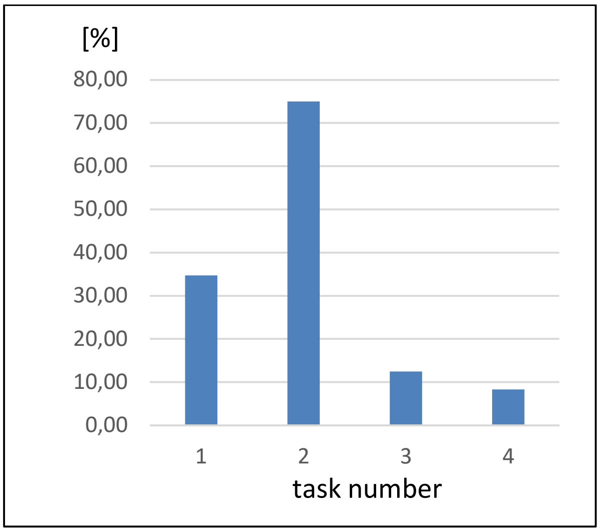
3 Testing pupils

The test was primarily focused on determining the success rates of primary school pupils in solving basic types of combinatorial tasks. Another output of the test was the information on the manner of solving these tasks. We wanted to get an idea of what strategies pupils use to solve combinatorial tasks. Altogether, 72 pupils participated in testing, of which 48 girls and 24 boys. Their age ranged from 10 to 11 years. Testing took place at six different elementary schools. Twelve pupils were selected from each school by a stratified selection. The pupils with specific learning disabilities were not selected and the sample was homogeneous in terms of pupils skills in mathematics. The test contained four open-answer tasks. The complete assignment of all four tasks is shown in Table 2. The tasks used were selected on the basis of a series of pretests to reduce undesirable effects that could adversely affect test results. Tasks were contextually different and more strategies could be used to solve them.

Task No.	Assignment
1.	A dodgeball tournament is held at the school. Five teams from five classes (3 A, 3 B, 4 A, 4 B and 5 A) were registered. In the tournament, all teams will play each other (each with each one once). How many matches will there be in total?
2.	Jindra has green and red socks in his drawer. Without looking, he took three socks out of the drawer. Can he be sure that he took out two socks of the same color?
3.	The teacher is ill and therefore the schedule changes on Monday. The pupils will have the following subjects: Czech language, mathematics, physical education, English language and music. The second lesson must be the English language and the fifth lesson physical education. What might the schedule be like on Monday?
4.	Maruška has to pay 11 CZK. She has only five-crown coins, two-crown coins and one-crown coins in her wallet. Find all possible ways of paying this amount when there are a lot of coins and she can use all kinds of coins or just some.

Tab. 2. Tasks assignment in the test.

The success of the solution has been evaluated for each task separately. Only the complete solution with all the options stated was considered to be correct. Graph 3 illustrates the percentage success of pupils in solving individual tasks.



Graph 3. Percentage success of pupils in solving individual tasks.

Task number 2 had the highest solution success rate. It was correctly solved by 77% of pupils. In many cases, the pupils correctly substantiated their claims, largely using a picture. Two-thirds of pupils who responded correctly drew all variants of the solution. Task number 1 was solved correctly by 34% of pupils. The pupils who used a spreadsheet or graph depiction of the break-down of all matches were more successful in solving the task. For the remaining two tasks, the success rate was 12.5% for the third task and 8% for the fourth task. It was the fourth task where the pupils achieved the smallest success rate. The pupils were unable to find all the options for solving the task. Most often, 4 or 7 options were stated as the correct answer. In both cases, this answer was stated by 18% of pupils. Only six pupils found all 11 options and they all used systematic search to do so.

The evaluation of available data shows that the pupils chose the following solution strategies to solve combinatorial tasks: using the table to find all solutions (Tasks 1 and 3); using the color picture and visualization of the situation (Tasks 1 and 2); using systematic recording when searching for all options (Tasks 3 and 4). In each case, the node graph (Task 1) and symbolic recording (Task 2) were successfully used. Figures 1 through to 4 show specific pupils solutions using different strategies.

Task 1. A dodgeball tournament is held at the school. Five teams from five classes (3 A, 3 B, 4 A, 4 B and 5 A) were registered. In the tournament, all teams will play each other (each with each one once). How many matches will there be in total?

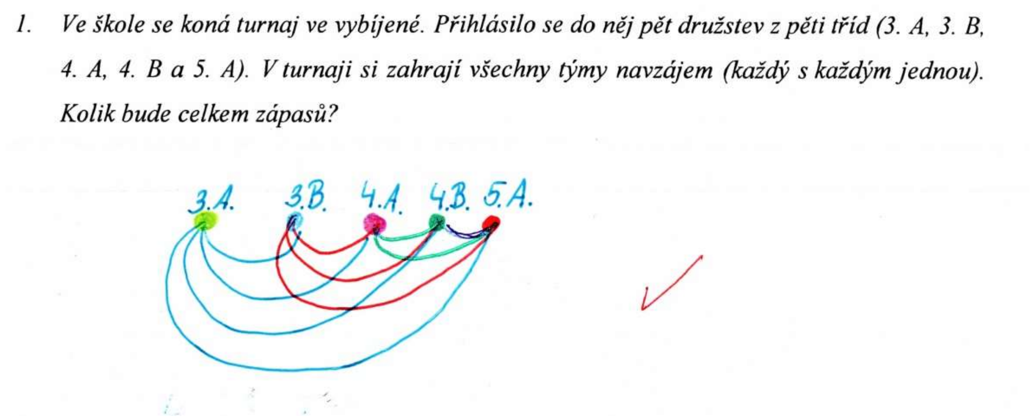


Fig. 1. Pupils solution of Task 1, use of visualization and node graph.

Task 2. Jindra has green and red socks in his drawer. Without looking, he took three socks out of the drawer. Can he be sure that he took out two socks of the same color?

Yes
either he took out
a) 2 red socks and 1 green sock, or
b) 2 green socks and one red sock

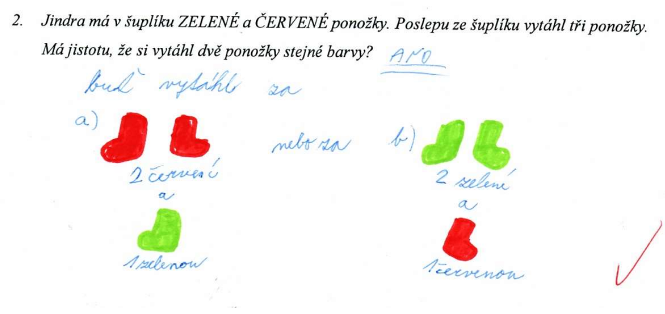
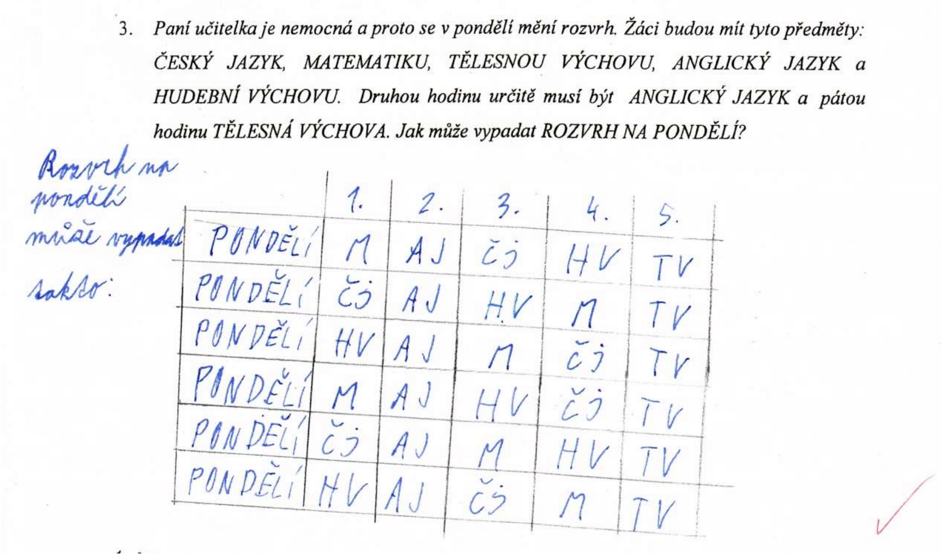


Fig. 2. Pupils solution of Task 2, use of visualization.

Task 3. The teacher is ill and therefore the schedule changes on Monday. The pupils will have the following subjects: Czech language, mathematics, physical education, English language and music. The second lesson must be the English language and the fifth lesson physical education. What might the schedule be like on Monday?

The answer was: "The Monday schedule might look like this."


3. *Paní učitelka je nemocná a proto se v pondělí mění rozvrh. Žáci budou mít tyto předměty: ČESKÝ JAZYK, MATEMATIKU, TĚLESNOU VÝCHOVU, ANGLICKÝ JAZYK a HUDEBNÍ VÝCHOVU. Druhou hodinu určitě musí být ANGLICKÝ JAZYK a pátou hodinu TĚLESNÁ VÝCHOVA. Jak může vypadat ROZVRH NA PONDĚLÍ?*

Rozvrh na pondělí může vypadat takto:

	1.	2.	3.	4.	5.
PONDĚLÍ	M	AJ	ČJ	HV	TV
PONDĚLÍ	ČJ	AJ	HV	M	TV
PONDĚLÍ	HV	AJ	M	ČJ	TV
PONDĚLÍ	M	AJ	HV	ČJ	TV
PONDĚLÍ	ČJ	AJ	M	HV	TV
PONDĚLÍ	HV	AJ	ČJ	M	TV

Fig. 3. Pupils solution of Task 3, use of spreadsheet.

Task 4. Maruška has to pay 11 CZK. She has only five-crown coins, two-crown coins and one-crown coins in her wallet. Find all possible ways of paying this amount when there are a lot of coins and she can use all kinds of coins or just some.

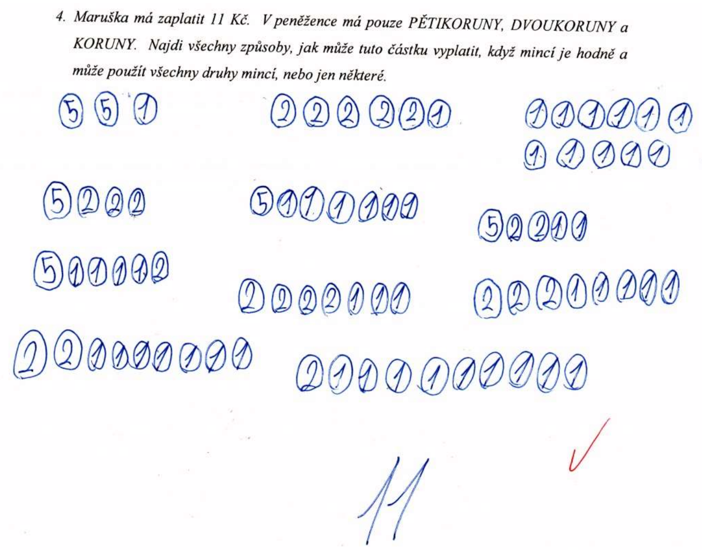

4. *Maruška má zaplatit 11 Kč. V peněžence má pouze PĚTIKORUNY, DVOUKORUNY a KORUNY. Najdi všechny způsoby, jak může tuto částku vyplatit, když mincí je hodně a může použít všechny druhy mincí, nebo jen některé.*

Fig. 4. Pupils solution of Task 4, use of systematic writing.

Conclusion

In our survey we looked into the results of a questionnaire survey focused on primary school teachers and their approach to including and solving combinatorial tasks in teaching mathematics. The available data clearly show that 48% of the participating teachers seldom or never assign combinatorial tasks to pupils. The analysis of the questionnaire also points to a certain correlation between the frequency of combinatorial tasks assignment and the textbooks used by teachers during tuition. It is clear from the questionnaire evaluation that teachers use three methods to solve combinatorial tasks with pupils: list of elements, consideration and graphical representation, and trial and error strategies. Consideration and graphical representation are employed even in over 70% of solutions.

The second part of the survey was the determination of success rate of primary school pupils in solving the basic types of combinatorial tasks and their method of solving. For this purpose, a test was created that included four typological and contextually different tasks. Each task was evaluated separately, both in terms of success and in terms of the use of solution strategies. Solely the complete solution, stating all the options, was considered to be correct. The highest success rate was achieved by pupils in the second task, which was 75%. They did quite well in solving the first task with 35% success rate. For the remaining tasks, success rate was small, only 12.5% and 8%. Pupils successfully used different strategies to solve the tasks: spreadsheet, systematic listing of all options, visualization using color pictures or node graph. The most effective strategy to find solutions is to use systematic recording when pupils create possible groups of elements through a logical process (for example, in Task 4 they determine the first correct possibility to use eleven one-crown coins and then gradually interchange coins so that the sum still equals 11). Similar findings are also reported by English in his study (1991).

The results of the surveys allow us to focus more in the following research on the creation of those activities that develop pupils logical and combinatorial thinking and that bring greatest difficulty in solution to pupils. We will also pay attention to the use of a larger variety of solving strategies. Last but not least, we would like to use these activities to motivate teachers to include the combinatorial tasks more often in teaching.

The results of the research will be farther used to improve the educational process of future elementary school teachers at the Faculty of Science, Humanities and Education at the Technical University of Liberec. They will be particularly used to promote constructive approaches in teaching mathematics. The teachers will be also offered specific and proven problem tasks which they can use in their practice.

Acknowledgement:

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