BASIC PRINCIPLES OF ORTHOGONAL AXONOMETRY WITH THE USE OF GEOGEBRA

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Abstract. The contribution introduces the use of dynamic applets created in GeoGebra in teaching the orthogonal axonometry for students of the third year of a Bachelor's degree Mathematics with a focus on Education. The dynamic applets are used with a big advantage in teaching for students with a lower level of spatial imagination, or for part-time students. The presented dynamic applets show chosen basic principles of the orthogonal axonometry.

Keywords: teaching, orthogonal axonometry, GeoGebra, dynamic applets

Mathematics Subject Classification: Primary 97B50, 68U05; Secondary 51N05

1 Introduction

Besides all, the students of the third year of a Bachelor's degree Mathematics with a focus on Education, Faculty of Science, Humanities and Education, Technical University of Liberec study the course Geometry 2. There are taught four themes – axiomatic principles of the spatial geometry, stereometry, Monge projection, and orthogonal axonometry - in this course. Unfortunately, at least half of the students of every year has got troubles with studying this course. A lower level of the spatial imagination and insufficient experience of solving spatial problems from the high schools are the main reasons for their troubles in studying the course Geometry 2. As a result, the students are lagging behind in creating images of spatial situations and problems, and consequently, they are mostly not able to solve the spatial problems correctly.

It is possible to show some appropriate spatial situations using models during the lectures and lessons. On the contrary, there are some spatial situations that are not easy to model them. E.g. the constructions in orthogonal axonometry are more difficult for modelling. For that reason, we started to create the dynamic applets in GeoGebra which bring the given spatial situations closer.

2 The dynamic applets created in GeoGebra

GeoGebra is the freeware mathematical dynamic software. Using various tools of GeoGebra, the users are able to create many different and useful dynamic applets. The applets can be dynamic either due to the main tool of the program – the slider or due to the free objects (points, lengths of line segments, radii of circles, …) constructed "freely" in GeoGebra.

The slider also allows the connection between the texts inserted in one 2D window, the objects drawn in the second 2D window, and objects constructed in the 3D window. Consequently, it is possible to show either basic principles, or constructive problems drawn in the 2D window and also in the 3D window at the same moment. The users can move or rotate with the spatial images in the 3D window by pressing the right button of the mouse and by dragging the mouse at once. That´s why the spatial images can be seen from different angles of view. As a result, the users can set the position of the spatial image under such an angle of view that fits best to them and in which they are able to see and understand the spatial situation best.

The green buttons titled "*Parallel view onto the spatial situation*" and used in the below-introduced applets are set by the authors under such an angle of view that fits best to them. The settings of the buttons are not able to be changed by the users.

2.1 Sharing of the dynamic applets created in GeoGebra

If the dynamic applets are ready, the task is how to use them in the teaching process. The teacher is able to use the GeoGebra source files of the dynamic applets during the lectures or lessons. On the other hand, there is a question if the students can work with the applets themselves during the lessons or at home. The answer is positive. There are at least two possibilities how the students gain the created dynamic applets. The teacher sends (e.g. via email) the GeoGebra source files of the dynamic applets to the appropriate group of the students or the teacher exports the dynamic applets into his GeoGebra account on the GeoGebra web page [1], see Fig. 1.
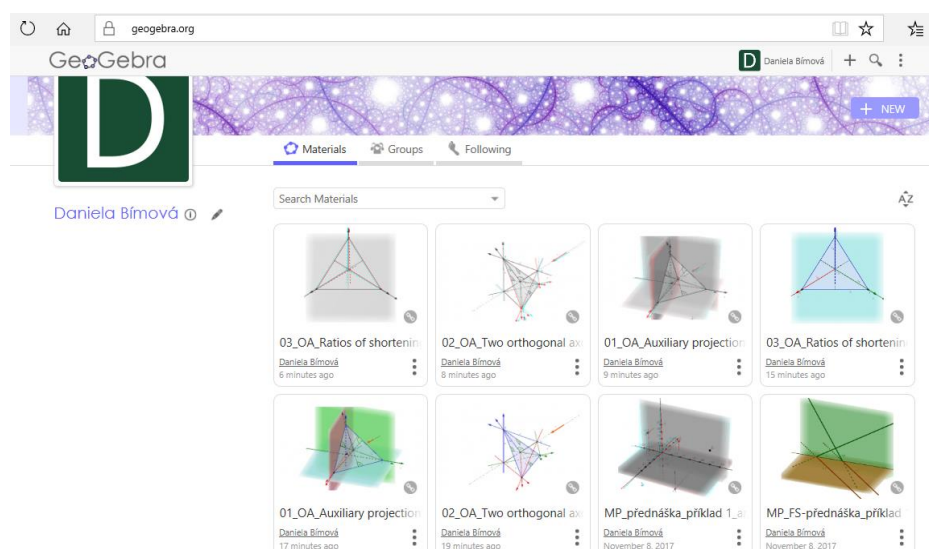
Fig. 1. Overview of the dynamic applets inserted on the GeoGebra web page of the author.

2.1.1 Kinds of sharing the dynamic applets on the GeoGebra web page

There are three different possibilities of sharing the dynamic applets created in GeoGebra on the GeoGebra web page:
* public visibility – everyone can find and view the dynamic applet;
* shared with link – only users who have a link can view the dynamic applet;
* private – only the author of the applet can find and view it.

Before setting the sharing of the dynamic applets as the "public visibility", it is better to share them with a link. In this case, the teacher has got the opportunity to send the links only to chosen groups of students and after that to test the functions and usefulness of the applets together with the students in the teaching process. If all the things work perfectly in the applets, the teacher can set the sharing of the applets as "public visibility".

2.2 Work with the dynamic applets created in GeoGebra

If the lessons of the course Geometry 2 take place in a PC laboratory, every student can find the appropriate applet through the usage of the link typed in the web browser on the computer. If the lessons take place in the classroom, the tablets can be used. Only the internet connection is needed. On the contrary, the internet connection is widespread on all the university campus of TUL. The students could also use their own electronic types of equipment.

GeoGebra was generated for various kinds of electronic equipment such as for PCs, tablets, or for mobile phones. It was also generated for different systems such as i.e. for Windows, Mac, Linux, or Android. For that reason, the dynamic applets can be used by the students in a lot of different electronic equipment. Let´s say, it can be used everywhere where there is the internet connection and it can be used by all the users that have the link. It means, every student should have the opportunity to find out the dynamic applets if he has obtained the links.

The work with the dynamic applets created in GeoGebra is quite intuitive. *Checkboxes, buttons,* or *sliders* help students how to follow the process of construction or solution in the dynamic applets. There are introduced three dynamic applets created in GeoGebra concerning the basic principles of orthogonal axonometry in the next section.

3 Dynamic applets of some basic principles of orthogonal axonometry in GeoGebra

3.1 Basic elements of the orthogonal axonometry

Orthogonal axonometry is a drawing method in which there is orthogonally drawn onto axonometric projection plane and onto three auxiliary projection planes – the top-view, the front-view, and the side-view auxiliary projection planes. It is true that all three auxiliary projection planes are orthogonal to each other. We can coincide the top-view projection plane with the coordinate plane (*xy*), further the front-view projection plane with the coordinate plane (*yz*), and finally the side-view projection plane with the coordinate plane (*xz*). Next, it must be true that the axonometric projection plane is concurrent with all three auxiliary

planes. It intersects the axes *x*, *y*, and *z* in a row in points *X*, *Y*, and *Z*. The triangle *XYZ* is called an axonometric triangle. The orthogonal projections x_a, y_a, z_a of the coordinate axes *x*, *y*, *z* are the heights of the triangle *XYZ*. The point of intersection O_a of the heights of the triangle *XYZ* is the orthogonal projection of the origin of the Cartesian coordinate system {*O, x, y, z*} onto the axonometric plane. Above-mentioned, the axonometric triangle *XYZ* lies in the axonometric projection plane.

There are shown all the mentioned basic elements of the orthogonal axonometry in Fig. 2. Fig. 2 displays the dynamic applet available on the GeoGebra website using the link [2].
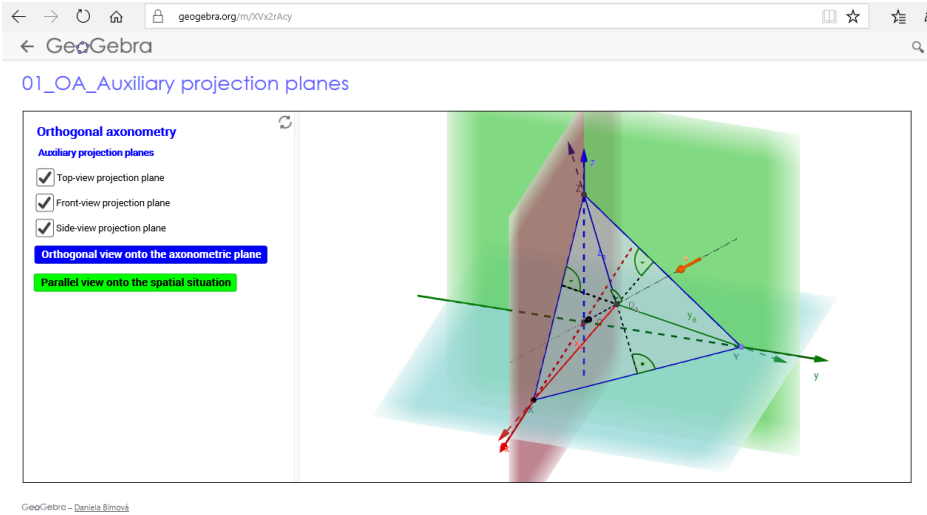

Fig. 2. Figure of the dynamic applet containing basic elements of the orthogonal axonometry.

The dynamic applet drawn in Fig. 2 consists of two windows. There is the 2D window on the left side and the 3D window on the right side of the Fig. 2. Besides of the titles, three checkboxes, that show or hide the particular auxiliary planes in the 3D window, and two buttons, pressing them, the direction of the point of view onto the spatial image is changed, are in the 2D window. The set "*Parallel view onto the spatial situation*" by the author is shown in the 3D window in Fig. 2. The preview of the dynamic applet in "*Orthogonal view onto the axonometric plane*" in a version for anaglyphic glasses is displayed in Fig. 3. The applet in version for anaglyphic glasses can be found on GeoGebra website by the link [3].
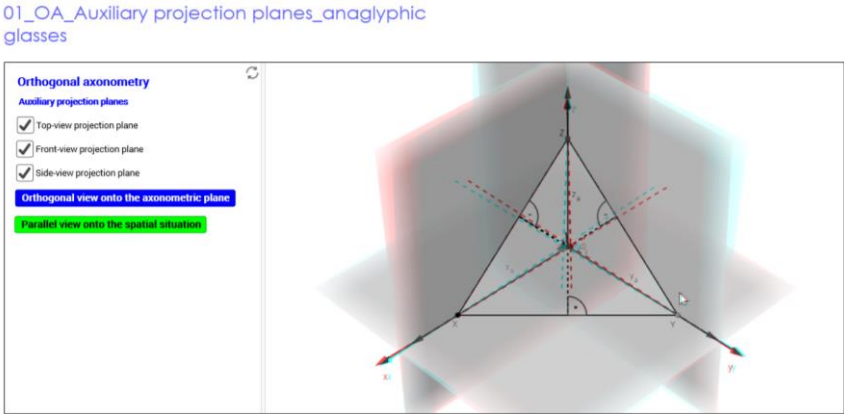

Fig. 3. The preview of the dynamic applet in the version for anaglyphic glasses.

3.2 Two orthogonal axonometries

In general, the axonometric projection plane coincides with the drawing plane. If there is given an acute-angle triangle *XYZ* in the drawing plane, two orthogonal axonometries exist for which the given triangle *XYZ* is the axonometric triangle. Further, if there are given three straight-lines passing through one point in such a way that every of the given straight-lines passes through the obtuse angle of the rest two straight-lines, two orthogonal axonometries exist for which three given straight-lines are the straight-lines of the axonometric axial cross.

It means that there are two different Cartesian coordinate systems $\{O, x, y, z\}$ and $\{O', x', y', z'\}$ which axes have their axonometric projections $x_a \equiv x'_a,\ y_a \equiv y'_a,\ z_a \equiv z'_a$ in the heights of the triangle *XYZ*. The points *O, O'* are symmetrical according to the plane *XYZ*.

The preview of the dynamic applet displaying the above described spatial situation is shown in Fig. 4. The dynamic applet itself can be opened using the web link [4].
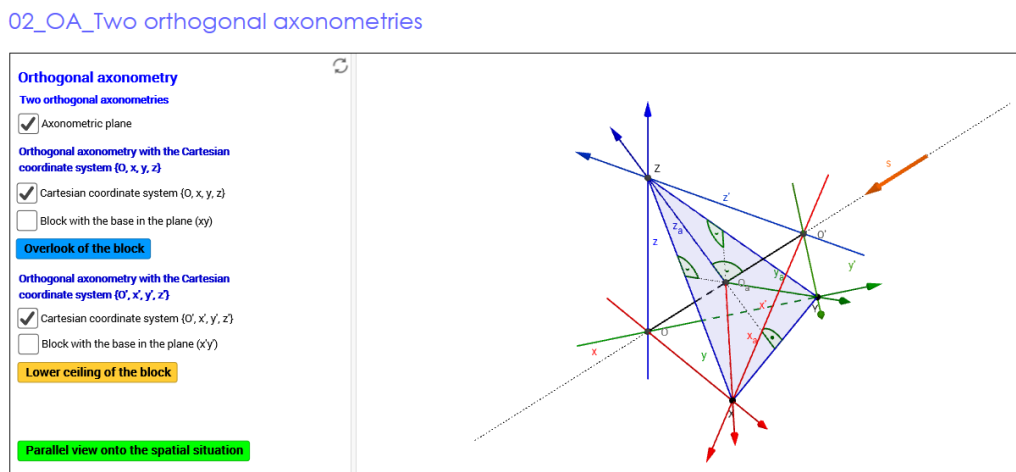

Fig. 4. The preview of the dynamic applet showing two Cartesian coordinate systems corresponding to one axonometric triangle *XYZ*.

We can imagine the axonometric projection plane as the vertical plane, after that one of the points *O, O'* is situated in front of the axonometric plane in the space. It is the point *O'* in our case. In the same half-space, there is also the observer. Follows, the second point is set behind the axonometric plane in the space. It is the point *O*.

Let's imagine that there is situated a block in each of two given Cartesian coordinate systems. If we think of the Cartesian coordinate system $\{O, x, y, z\}$ and if we look at the spatial situation orthogonally in the direction to the axonometric projection plane, we see the overlook of the given block. See Fig. 5. On the contrary, if we think of the Cartesian coordinate system $\{O', x', y', z'\}$ and if we look at the spatial situation orthogonally in the direction to the axonometric projection plane, we see the lower ceiling of the given block. In Fig. 6 there is displayed the lower ceiling of such a block.

Both the Cartesian coordinate systems together with the blocks are incorporated into one dynamic applet available using the above-mentioned link [4].
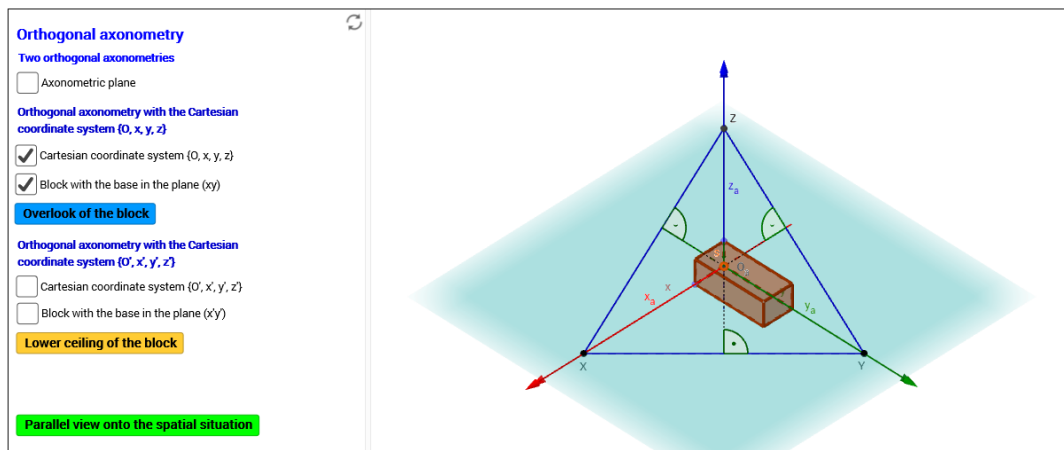
Fig. 5. The overlook of the block with one of its bases lying in the plane (*xy*)
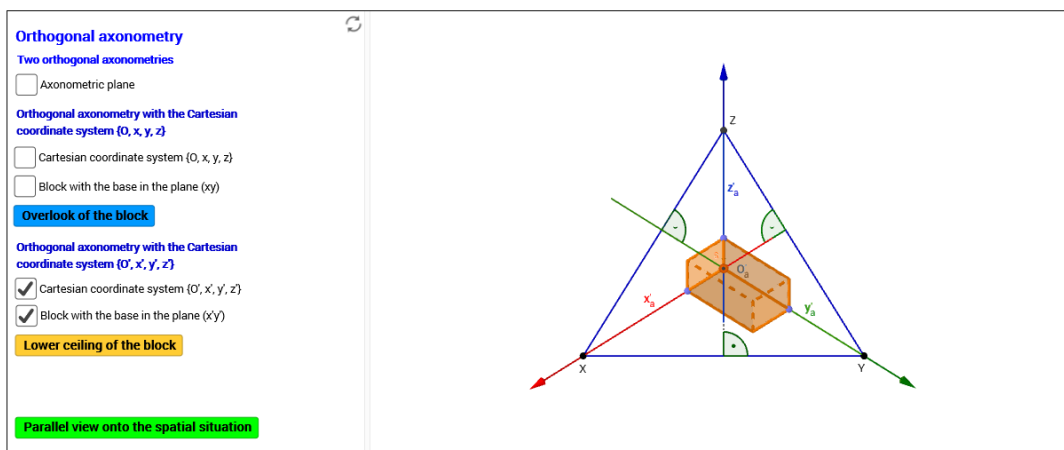

Fig. 6. The lower ceiling of the block with one of its bases lying in the plane (*x´y´*)

There are used five checkboxes in the applet. The corresponding objects of the spatial situation are interconnected with the appropriate checkbox. Activating the particular checkbox, the interconnected objects appear on the right side of the applet in the 3D window. The checkbox is activated if the check mark is in it. See e.g. Fig. 6. Further, there are two buttons - "*Overlook of the block*" and "*Lower ceiling of the block*" – that enable to switch between the overlook and the lower ceiling views of the block. The third button of the applet – "*Parallel view onto the spatial situation*" is programmed in such a way to all the drawn objects rotate into the set position in which the spatial situation is better seen according to the author.

The dynamic applet in the version for anaglyphic glasses was also generated for displaying two orthogonal axonometries with one corresponding axonometric triangle *XYZ*. It can be found out using the web link [5]. Fig. 7 shows the preview of "*Parallel view onto the spatial situation*" of overlooking of the block in the version for anaglyphic glasses.
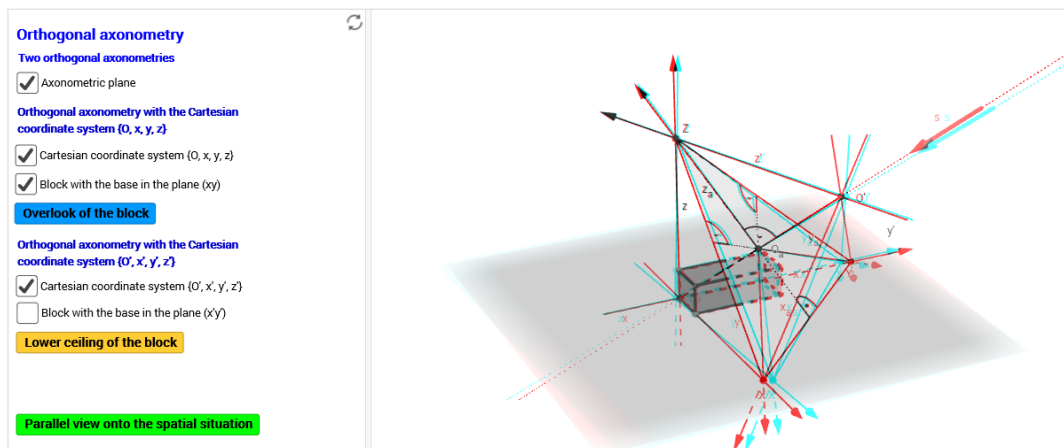
Fig. 7. Parallel view onto the spatial situation of overlooking of the block in the version for anaglyphic glasses.

3.3 Displaying the units on the axonometric projections of the coordinate axes

In comparison with e.g. the Monge projection, it is not so easy to construct projections of a point that is given by the coordinates in orthogonal axonometry. It is needful to know how the units lying on the particular axes x, y, and z are projected onto the axonometric projections x_a, y_a, z_a of the axes x, y, and z. It means we are looking for ways of displaying the units on the axonometric projections of the particular axes. The task is to find out the axonometric projections $A_a \in x_a$, $B_a \in y_a$, $C_a \in z_a$ of the points $A \in x$, $B \in y$, and $C \in z$ of the Cartesian coordinate system $\{O, x, y, z\}$ in which the lengths of the line segments OA, OB, and OC are of the same unit.

We can find out the points A_a, B_a, C_a in two ways.

Firstly, the axonometric projections $A_a \in x_a$, $B_a \in y_a$, $C_a \in z_a$ of the points $A \in x$, $B \in y$, and $C \in z$ can be constructed by rotation of the auxiliary planes round the lines of intersection between auxiliary projection planes and axonometric projection plane into the axonometric projection plane. We put the unit in its true length on the rotated axes x_0, y_0 in the axonometric projection plane. As a result, we receive the points A_0, B_0 on the axes x_0, y_0. It is true that $|A_0 O_0| = |B_0 O_0| = j$. Using the planar axial affinity, we construct the axonometric projections A_a, B_a of the points A, B on the axonometric projections x_a, y_a of the axes x and y. For better imagination, the reader can see Fig. 10.

Secondly, the axonometric projections $A_a \in x_a$, $B_a \in y_a$, $C_a \in z_a$ of the points $A \in x$, $B \in y$, and $C \in z$ can be drawn by folding of the plane passing through the corresponding axis and its axonometric projection. In Fig. 11 there is shown the step by step process of folding of the plane (zz_a) into the axonometric projection plane. The unit j is put on the folded axis (z) in its true length in the axonometric projection plane. Consequently, we receive the point (C) on the axis (z) for which it is true that $|(O)(C)| = j$. Using the planar axial affinity, we draw the axonometric projection C_a of the point C onto the axonometric projection z_a of the axis z.
Both the described ways of constructions are inserted into one dynamic applet created in GeoGebra. There are three parts in this dynamic applet. On the left side, there is one of the 2D

windows. The user can find two checkboxes in this 2D window after opening the applet. Each checkbox corresponds to one way of the above-described constructions. Next, three buttons for changing the angle of view onto the spatial situation are at the bottom of the left 2D window. The second 2D window is situated in the middle of the applet. There are drawn constructions in orthogonal axonometry in this middle 2D window. On the right side, there is the 3D window in which 3D images of the axonometric constructions are displayed. See Fig. 8.
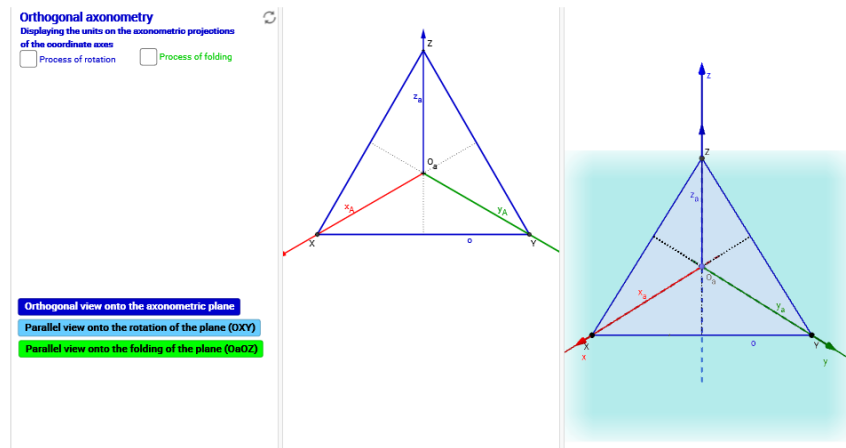

Fig. 8. Preview of the dynamic applet created in GeoGebra showing two ways of the constructions of displaying the units on the axonometric projections of the coordinate axes after its opening.
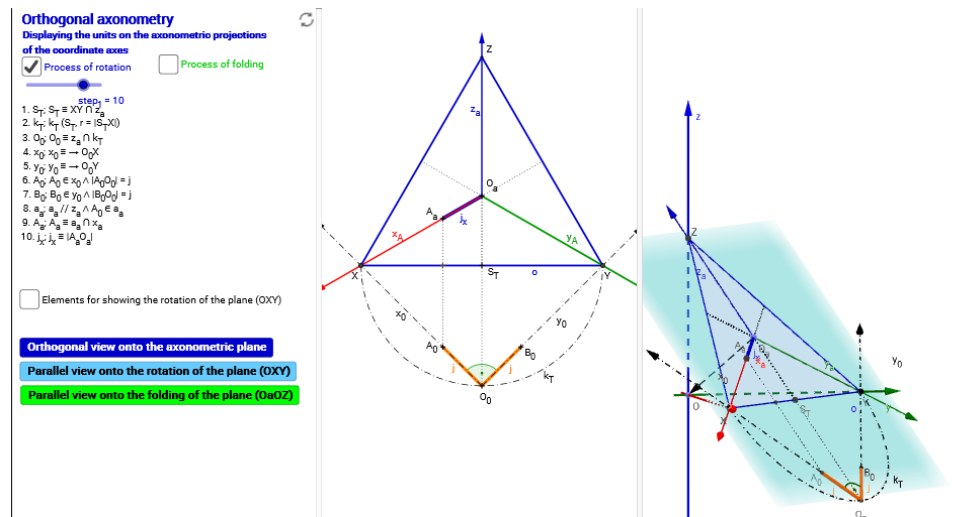

Fig. 9. The tenth step of the process of the construction using the way of rotation.

In this GeoGebra applet, there is used the abovementioned connection between 2D, 3D constructions and symbolic construction entries. The objects in 2D, 3D constructions and the particular steps of the symbolic construction entries are connected together with the usage of the dynamic tool of GeoGebra – with the slider Moving the slider, the particular steps of the

symbolic construction entries appear in the left 2D window and the corresponding 2D and 3D objects appear in the middle 2D window and in the right 3D window in one moment. The tenth step of the process of construction of the axonometric projections A_a, B_a of the points A, B on the axonometric projections x_a, y_a of the axes x, y is shown in Fig. 9..

Activating the checkbox "*Elements for showing the rotation of the plane (OXY)*", the objects for showing the process of rotation in 3D construction appear. Moving the point O_0, the plane (OXY) rotates with all the objects around the straight-line XY. See Fig. 10.
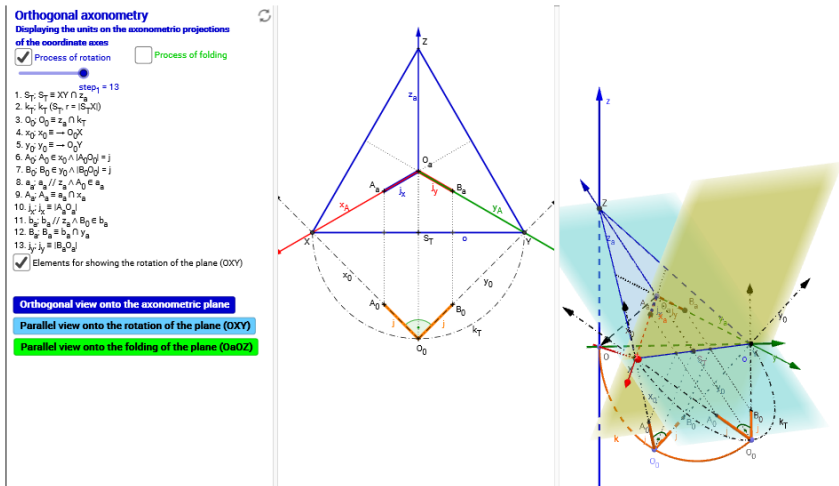

Fig. 10. Rotation of the plane (OXY) around the axis XY in 3D construction.

The similar process of solution is created also for the second way of displaying the units on the axonometric projections of the coordinate axes - for the process of folding - in the GeoGebra applet. Both ways of constructions are drawn and described by the symbolic construction entries in Fig. 11.
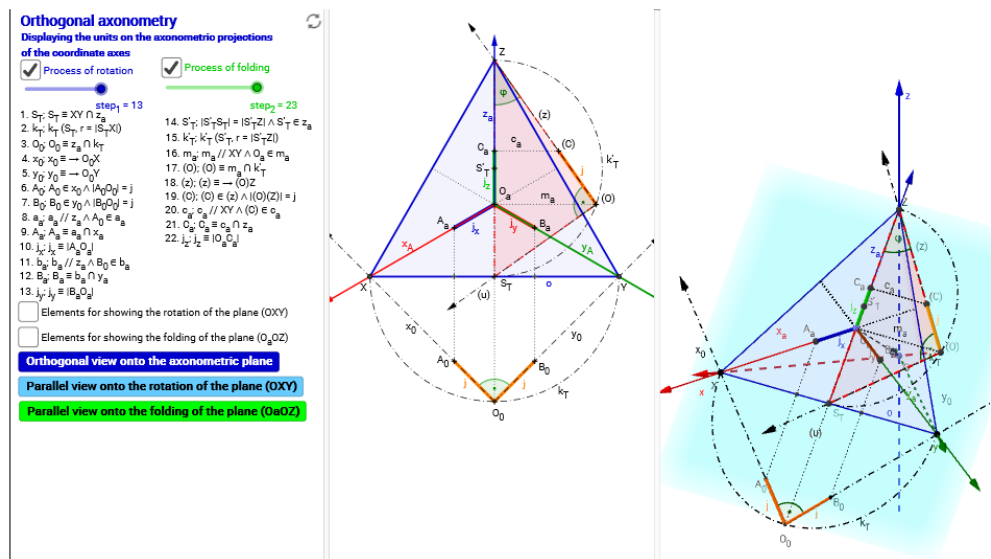

Fig. 11. Constructions of both the ways of displaying the units on the axonometric projections of the coordinate axes.

The possibility of showing only the construction of folding the plane (zz_a) also exists. The checkbox „*Process of rotation*" must be deactivated in this case.

The above-mentioned applet can be opened via the web link [6]. The same applet generated in the version for anaglyphic glasses is possible to found on the web link [7].

4 Conclusion

In summary, we are able to say on the basis of our experiences that the best way of teaching the various kinds of projection methods of the descriptive geometry (e.g. orthogonal axonometry) is the combination of classical interpretation of the principles and problems of the projection method using a marker and a whiteboard, of electronic presentation containing a text and static images of spatial situations and solved problems, and of using the e-learning material. Dynamic applets created in GeoGebra can be used with a big advantage as the e-learning material that helps students to see better the spatial situations and that shows students the step by step processes of solving the spatial problems. The dynamic applets are very useful especially for part-time students of a Bachelor's degree Mathematics with a focus on Education during their study at home.

References

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