Slovak University of Technology in Bratislava
Faculty of Mechanical Engineering
17th Conference on Applied Mathematics **APLIMAT 2018**

Proceedings

ON NORMAL ASSUMPTIONS ON DEMAND FUNCTION AND ITS ELASTICITY

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Abstract. In this note we consider the demand function $D = D(p)$, where p is price of a certain good and we introduce some natural assumptions on D in terms of the corresponding elasticity coefficient. We derive some properties of D which follow from the normal assumptions, and we provide economic interpretation and application. We also present some classes of of functions which satisfy the normal assumptions.

Keywords: Elasticity of function, demand function, mathematical economics, mathematical analysis

Mathematics subject classification: Primary 34E15, 34C25, 35M10; Secondary 49J45

1 Introduction

Economist often study how demand for a certain commodity reacts to a change of price. It can be measured by how many units the quantity demanded will change per unit of money increase in price. However, there is unsatisfactory aspect to this way of measuring the sensitivity of demand to price changes because one unit of money increase in the price may be considerable for some commodities (bread or milk, for instance) whereas it can be insignificant for other commodities (such as cars). This difficulty can be eliminated if relative changes are used instead. It can be asked by what percentage the quantity demanded changes when the price increases by 1 percent. The number obtained in this way is independent of the units in which both quantity and price are measured. This number is called the price elasticity of demand, measured at a given price. Assume now that the demand for a commodity can be described by the function $x = D(p)$, where p denotes the price. The change of price from p to $p + \Delta p$, yields the change of quantity of demand, the absolute change of x being $\Delta x = D(p + \Delta p) - D(p)$. Hence, the relative (or proportional) change of x is given by $\frac{\Delta x}{x} = \frac{D(p+\Delta p)-D(p)}{D(p)}$. The ratio of two aforementioned relative changes equals to

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{p}{D(p)} \frac{D(p + \Delta p) - D(p)}{\Delta p} .$$

If $\Delta p = \frac{p}{100}$, the parameter p increases by 1 percent, and we get

$$\frac{\frac{\Delta x}{x}}{\frac{\Delta p}{p}} = \frac{\Delta x}{x} \cdot 100 ,$$

whereby the right-hand side in the equation above represents the change of quantity of demand measured in terms of percentages of x. This quantity is called the average elasticity of x in the interval

$[p, p + \Delta p]$, and it depends both on the price change Δp and on the price p, but is unit free. Therefore it makes no difference whether the quantity of demand is measured in tons, kilograms or whatever units, or if the price is measured in dollars, euros, or any other currency. To define the actual elasticity of $D(p)$ with respect to p (denotes by $E_p D(p)$) in a manner that it does not depend on the amount of increase in p, we consider the limit as $\Delta p \longrightarrow 0$, getting

$$E_p D(p) := \frac{p}{D(p)} \cdot D'(p)$$

Since the increase of the price p for normal goods results in decrease in the demand $D(p)$, we expect $E_p D(p)$ to be negative for any admissible p.

The price elasticity of demand varies enormously from one commodity to another. The more substitutes there are for a commodity and the closer they are, the greater will be the price elasticity of demand (ignoring the negative sign). The reason is that consumers will be able to switch to the substitutes when the price of the commodity increases. For example, the price elasticity of demand for a particular brand of a product will probably be fairly high, especially if there are many other similar brands. By contrast the demand for a product in general should be less sensitive to a change in price. As well, the higher the proportion of consumers' income is spent on a certain commodity, the more they will have to reduce their consumption of it following a rice in price i.e. the more elastic will be the demand. Economists are interested in knowing the sensitivity of demand not just to a change in price but also to changes in other variables such as consumers' incomes (income elasticity of demand) or expenditure on a particular advertising campaign or other forms of promotion. Furthermore, they consider elasticities of supply, elasticities of revenue, cost elasticity and several other kinds of elasticity. It is therefore logical and certainly helpful to define elasticity for a general differentiable function $f = f(x)$.

- If $|E_x f(x)| > 1$, then f is elastic at x,
- If $|E_x f(x)| = 1$, then f is unit elastic at x,
- If $|E_x f(x)| < 1$, then f is inelastic at x,
- If $|E_x f(x)| = 0$, then f is perfectly inelastic at x,
- If $|E_x f(x)| = +\infty$, then f is perfectly elastic at x.

If the demand is price elastic, quantity demanded changes proportionately more than price, therefore a rise in price will lead to a reduction in consumers' expenditure on the commodity and hence to a reduction in the total producers' revenue. When the demand is price inelastic, price changes proportionately more than quantity demanded, so a rise in price will lead to an increase in total expenditure and revenue. If the price elasticity of demand is unity then the quantity demanded changes by the same proportion as the price. There are some rules for elasticities of sums, products, quotients and composite functions might be useful. It can easily be shown that if $f = f(x)$ and $g = g(x)$ are positive valued differentiable functions of x and A is a constant, then the following rules hold:

- $E_x A = 0$,
- $E_x(fg) = E_x(f) + E_x(g)$,

- $E_x(\frac{f}{g}) = E_x(f) - E_x(g)$,
- $E_x(f+g) = \frac{f}{f+g}E_x f + \frac{g}{f+g}E_x g$,
- $E_x(f-g) = \frac{f}{f-g}E_x f - \frac{g}{f-g}E_x g$,
- $E_x(f \circ g) = E_u f(u) \cdot E_x u$, where $u = g(x)$.

Also, we note that, if $y = f(x)$ admits an inverse function $x = f^{-1}(y)$, then it holds that $E_y f^{-1} = \frac{1}{E_x f}$. We also use the notation $E_{g,x}$ instead of $E_x f$, which is sometimes more convenient. Then we can re-state some of the following results as follows:

Proposition 1.1. If $x = x(p)$ (i.e., $p = p(x)$), it holds that:

(i) $E_{px,p} = 1 + E_{x,p}$,

(ii) $E_{px,x} = 1 + E_{p,x}$,

(iii) $E_{p,x}E_{x,p} = 1$.

2 Examples

Further, we provide six typical examples of demand functions (cf. [2]).

$$x = a - bp^c, a, b, c > 0 \tag{1}$$

$$x = a + bp^{-c}, a, b, c > 0 \tag{2}$$

$$x = \frac{b}{p+a}, a, b > 0 \tag{3}$$

$$x = \sqrt{a - bp}, a, b > 0 \tag{4}$$

$$x = ae^{-bp}, a, b > 0 \tag{5}$$

$$x = p^a e^{-b(p+c)}, a, b, c > 0 \tag{6}$$

The function defined by (1) is usually considered as a demand function for $c = 1$ (linear function), $c = 2$(quadratic function) and $c = \frac{1}{2}$ (square root). All of the functions (1)-(6) satisfy the standard assumptions on the demand functions, but, on top of that, also share the common feature (with the exception of (2)): for all admissible values of the price p determined by $\mathcal{D}(x) = \{p \geq 0 : x(p) \geq 0, x'(p) \leq 0\}$ it holds that the elasticity coefficient $E_{x,p}$ is a decreasing function of the price p, i.e., as the price p increases, the demand function monotonically makes then transition from the interval of non-elasticity to the interval of elasticity, or, for short, $\frac{dE_{x,p}}{dp} \leq 0$.

3 Demand Function and Revenue Function

Since the affine function is instructive and simple example, we describe the connection between in the case of the demand function (1). In such a case, we have

$$x = a - bp \quad \text{or} \quad p = \frac{a-x}{b} \quad \text{for } a, b > 0 \ .$$

The demand function is well-defined for $p \in [0, \frac{a}{b}]$. The elasticity coefficient is

$$E_{x,p} = -\frac{bp}{x} \ .$$

The inelasticity interval is $p \in [0, \frac{a}{2b}\rangle$, whereas the elasticity interval is $p \in \langle \frac{a}{2b}, \frac{a}{b}]$. Provided

$$p = \frac{a}{2b}\ , \quad x = \frac{a}{2}\ , \quad \text{it results} \quad E_{x,p} = -1\ .$$

We have

$$\frac{dE_{x,p}}{dp} = -\frac{ab}{x^2} \leq 0\ .$$

Then the function of (total) revenue reads $R(p) = px = ap - bp^2$, and it attains the maximal value

$$R^* = \frac{a^2}{4b} \quad \text{for} \quad p = \frac{a}{2b}\ , \quad x = \frac{a}{2}\ , \quad E_{x,p} = -1\ .$$

On the other hand, we can express the total revenue in terms of quantity of demand, getting

$$R(x) = \frac{ax - x^2}{b}\ ;$$

whereby the marginal revenue is

$$MR = R' = \frac{dR}{dx} = \frac{a - 2x}{b}\ .$$

If the marginal revenue equals zero, i.e., $R' = 0$, the total revenue admits stationary point $x^* = \frac{a}{2}$ and $p(x^*) = \frac{a}{2b}$. We have $R' > 0$ ($R' < 0$, resp.), i.e., the total revenue increases (decreases, resp.) for $0 \leq x \leq \frac{a}{2}$ ($\frac{a}{2} \leq x \leq a$., resp.). Thus, the maximum of the total revenue is attained at $x = x^*$. In terms of elasticity this means that demand is elastic for $0 \leq x \leq \frac{a}{2}$ and inelastic for $\frac{a}{2} \leq x \leq a$. Hence,

$$\frac{dE_{x,p}}{dx} = \frac{a}{x^2} \geq 0\ .$$

This provides as additional assumption on the demand function. That is to say, normal assumptions on the demand function follow from natural assumptions on the total revenue function $R = R(x) = px$. Indeed, we have:

$$\frac{dR}{dx} = \frac{R}{x}(1 + E_{p,x}). \tag{7}$$

We distinguish three cases:

(i) If $E_{p,x} \in \langle -1, 0\rangle$ then (7) we get $\frac{dR}{dx} \geq 0$. The interval where total revenue increases with respect to demand x coincides with the interval of elasticity of the demand function since it holds that $E_{x,p}E_{p,x} = 1$. We conclude:

$$E_{p,x} \in \langle -1, 0\rangle \Rightarrow E_{x,p} < -1\ .$$

(ii) If $E_{p,x} = -1$, then (by (7)) $\frac{dR}{dx} = 0$ for $x = x^*$ and the total revenue admits stationary point.

(iii) If $E_{p,x} < -1$ then $\frac{dR}{dx} \leq 0$. The interval where total revenue decreases with respect to demand x coincides with the interval of inelasticity of the demand function, i.e., $E_{x,p}E_{p,x} = 1$ yields

$$E_{p,x} < -1 \Rightarrow E_{x,p} > -1\ .$$

We recall that natural assumption on the total revenue function provides that for a small quantity of demand x the total revenue increases, which means that the demand grows faster then the price declines to some eventual stationary pointx^*, where the total revenue attained its maximum. After the point x^*, total revenue decreases. In terms of elasticity this means that the demand function which depends on the price p is decreasing. The consideration above allows us to formulate normal assumptions on the demand function:

Normal Assumptions on Demand Function:

$$p \geq 0 \ , \tag{8}$$
$$x \geq 0 \ , \tag{9}$$
$$x' \leq 0 \ , \tag{10}$$
$$\frac{dE_{x,p}}{dp} \leq 0 \ . \tag{11}$$

4 Elasticity of Demand Function

Definition 4.1 The elasticity coefficient of the second order of $x = x(p)$ is denoted by $E^{(2)}_{x,p}$ and it is the elasticity coefficient of the function $E = E_{x,p}$, i.e.,

$$E^{(2)}_{x,p} := E_{E,p} = \frac{p}{E}\frac{dE}{dp}. \tag{12}$$

Definition 4.2 The elasticity coefficient of order $k+1, k \in \mathbb{N}$ is defined by $E^{(k)} = E^{(k)}_{x,p}$, i.e.,

$$E^{(k+1)}_{x,p} := E_{E^{(k)},p} = \frac{p}{E^{(k)}}\frac{dE^{(k)}}{dp}. \tag{13}$$

Now we can rewrite condition (11), $\frac{dE_{x,p}}{dp} \leq 0$, coupled with the condition $E_{x,p} \leq 0$ in terms of the elasticity coefficient of the second order, getting

$$E^{(2)}_{x,p} \geq 0. \tag{14}$$

Further, we can rewrite the properties of typical demand functions in terms of higher-order elasticity coefficients.

1. The demand function (1)

$$x = a - bp^c, a, b, c > 0$$

is defined for $p \in [0, (\frac{a}{b})^{\frac{1}{c}}]$. Then it holds that:

$$\begin{aligned} E =& E_{x,p} = -bcp^c x^{-1} \leq 0 \ , \\ E^{(2)} =& E_{E,p} = c - E \ , \\ E^{(3)} =& E_{c-E,p} = -E \ , \\ E^{(4)} =& E_{-E,p} = c - E \ . \end{aligned}$$

Notice that we have $c > 0, -E \geq 0 \Rightarrow E^{(2)} = E_{E,p} = c - E \geq 0$, and so normal assumptions are fulfilled. If

$$p^* = (\frac{a}{b(c+1)})^{\frac{1}{c}}$$

we get $E_{x,p} = -1$, which is the price for which demand monotonically changes from inelasticity interval to the elasticity interval. On the other hand, total revenue increases from $p = 0$ do $p = p^*$, and at $p = p^*$ attains its maximal value, whereby for $p > p^*$ we have decrease of total revenue.

2. The demand function (3)

$$x = \frac{b}{p+a}, a, b > 0$$

is defined for $p \geq 0$.

$$\begin{aligned} E =& E_{x,p} = -b^{-1}px \leq 0 \ , \\ E^{(2)} =& E_{E,p} = 1 + E \ , \\ E^{(3)} =& E_{1+E,p} = E \ , \\ E^{(4)} =& E_{E,p} = 1 + E \ . \end{aligned}$$

This function is inelastic for any non-negative price p, i.e., $0 \geq E_{x,p} > -1$, so that $E^{(2)} = E_{E,p} = 1 + E > 0$.

3. The demand function (4)

$$x = \sqrt{a - bp}, a, b > 0,$$

is defined for $p \in [0, \frac{a}{b}]$.

$$\begin{aligned} E =& E_{x,p} = -0.5bpx^{-2} \leq 0 \ , \\ E^{(2)} =& E_{E,p} = 1 - 2E \ , \\ E^{(3)} =& E_{1-2E,p} = -2E \ , \\ E^{(4)} =& E_{E,p} = 1 - 2E \ . \end{aligned}$$

Since $-E \geq 0 \Rightarrow E^{(2)} = E_{E,p} = 1 - 2E \geq 0$, the normal assumptions are satisfied. For

$$p^* = \frac{2a}{3b}$$

we get $E_{x,p} = -1$, and so p^* is the price for which demand monotonically changes from inelasticity interval to the elasticity interval. On the other hand, total revenue increases from $p = 0$ do $p = p^*$, and at $p = p^*$ attains its maximal value, whereby for $p > p^*$ we have decrease of total revenue.

4. The demand function (5)

$$x = ae^{-bp}, a, b > 0,$$

is defined for $p \geq 0$. Computation gives

$$\begin{aligned} E =& E_{x,p} = -bp \leq 0 \ , \\ E^{(2)} =& E_{E,p} = 1 > 0 \ , \\ E^{(3)} =& E_{1,p} = 0 \ . \end{aligned}$$

Provided

$$p^* = \frac{1}{b} \ ,$$

we get $E_{x,p} = -1$ and so p^* is the price for which demand monotonically changes from inelasticity interval to the elasticity interval. On the other hand, total revenue increases from $p = 0$ do $p = p^*$, and at $p = p^*$ attains its maximal value, whereby for $p > p^*$ we have decrease of total revenue.

5. The demand function (6)

$$x = p^a e^{-b(p+c)}, a, b, c > 0$$

is defined for $p \geq 0$.
The elasticity coefficient is

$$E = E_{x,p} = a - bp$$

and so

$$E = E_{x,p} = a - bp \leq 0 \Rightarrow p \geq \frac{a}{b}.$$

Thus for $p \in [\frac{a}{b}, \infty\rangle$ we obtain

$$\begin{aligned} E^{(2)} =& E_{E,p} = -\frac{bp}{E} = 1 - \frac{a}{E} > 0 \ , \\ E^{(3)} =& E_{-\frac{bp}{E},p} = \frac{a}{E} \ , \\ E^{(4)} =& E_{\frac{a}{E},p} = -E_{E,p} = -E^{(2)} \ . \end{aligned}$$

For

$$p^* = \frac{a+1}{b}$$

we get $E_{x,p} = -1$, and so p^* is the price for which demand monotonically changes from inelasticity interval to the elasticity interval. On the other hand, total revenue increases from $p = \frac{a}{b}$ to $p = p^*$, and at $p = p^*$ attains its maximal value, whereby for $p > p^*$ we have decrease of total revenue.

Finally, we formulate natural requirements for the demand function with respect to its elasticity.

Normal assumptions on demand function in terms of elasticity

$$\begin{aligned} p \geq& 0 \ , \\ x \geq& 0 \ , \\ E_{x,p} \leq& 0 \ , \\ E^{(2)}_{x,p} \geq& 0 \ . \end{aligned}$$

Theorem 4.1 If $x = x(p), p, x \geq 0$, $x' \leq 0$, $x(0) > 0$ and

$$E^{(2)}_{x,p} = \alpha + \beta E, \alpha > 0, \beta \neq 0$$

then the function $x = x(p)$ satisfies normal assumptions on the demand function.
Proof. Starting from $E^{(2)}_{x,p} = \alpha + \beta E$, according to (12) we get

$$\frac{E'}{E(\alpha + \beta E)} = \frac{1}{p}$$

or, equivalently,

$$\frac{dE}{E(\alpha+\beta E)} = \frac{1}{p}dp.$$

The solution E of this ordinary differential equations is

$$\frac{E}{\alpha+\beta E} = cp^{\alpha}$$

which can be written in its explicit form as

$$E = \frac{\alpha c p^{\alpha}}{1-c\beta p^{\alpha}} \ . \tag{15}$$

Next, we have

$$\frac{x'}{x} = \frac{\alpha c p^{\alpha-1}}{1-c\beta p^{\alpha}}$$

whereby

$$x = b(1-c\beta p^{\alpha})^{-1/\beta} \ . \tag{16}$$

To begin with, since $x(0) > 0$, we get $b > 0$. Then it follows that $x \geq 0, b > 0 \Rightarrow 1 - c\beta p^{\alpha}$.
Case 1.
If it holds that $c\beta < 0$, then $1 - c\beta p^{\alpha} > 0$ for every $p \geq 0$.
From (15) and $x' \leq 0$ we deduce $c < 0$, and also from (15) we recover

$$E' = \frac{\alpha^2 c p^{\alpha-1}}{(1-c\beta p^{\alpha})^2} \ . \tag{17}$$

But then from $c < 0$ it results $E' \leq 0$.
Case 2.
If it holds that $c\beta > 0$, $\alpha > 0$ and $1 - c\beta p^{\alpha} \geq 0$ provide $0 \leq p \leq (c\beta)^{-\frac{1}{\alpha}}$, $x(0) = b > 0$.
Hence, $E \leq 0 \Rightarrow c < 0$. Now from $c\beta > 0$ and $c < 0$ we have $\beta < 0$. Therefore, for $\alpha > 0, \beta < 0$ we have$E^{(2)}_{x,p} = \alpha + \beta E \geq 0$.

Q.E.D.

Conclusion
One example of the class of demand functions which satisfy normal assumptions is:

$$x = b(1-c\beta p^{\alpha})^{-1/\beta} \ ,$$

where

$$b > 0, c < 0, \alpha > 0, \beta \neq 0.$$

5 General setting

In a broader context, our goal is to examine and describe the structure of elementary functions of one variable according to the properties of their generalized elasticity coefficients. For a given open interval $D \subset (0, +\infty)$ we introduce the class of functions X_D, defined by

$$X_D := \{g : D \longrightarrow (0, +\infty) : g^{(n)}(x) > 0, \ n \in \mathbf{N} \cup \{0\} \ , x \in D\} \ .$$

For $f, g \in X_D$ we set $f \approx g$ iff there exists $\lambda \in \mathbf{R}\backslash\{0\}$ such that $g = \lambda f$, $[g] := \{f \in X_D : f \approx g\}$, $\overline{X}_D := X_D/\approx$. Then $E : \overline{X}_D \longrightarrow X_D$, $E[g](x) := E_{g,x}$ is well-defined injection.

Definition 5.1. We say that function g is

(i) φ_n-type recursive function if there exists $n_0 \in \mathbf{N}$ such that for every $n \in \mathbf{N}$, $n \geq n_0$, it holds that $E_g^{(n+1)} = \varphi_n(E_g^{(n)})$; if for some φ it holds that $\varphi_n = \varphi$ for every $n \geq n_0$, we say that g (where $g = g(\varphi)$) is φ-type recursive function,

(ii) is n_0-static, if φ_n-type recursive function g satisfies $\varphi_n(\xi) = \xi$ for every $n \geq n_0$.

Example 5.1. If we set $n_0 := 2$ and

$$\varphi_n(\xi) := \begin{cases} -\xi, & \text{if } n \geq 3 \text{ is odd} \\ c - \xi, & \text{if } n \text{ is even,} \end{cases}$$

$$\left(\psi_n(\xi) := \begin{cases} \xi, & \text{if } n \geq 3 \text{ is odd} \\ 1 + \xi, & \text{if } n \text{ is even,} \end{cases} \text{resp.}\right)$$

then $g(x) := a - bx^c$ ($g(x) := b(x-a)^{-1}$, resp.) is φ_n-type (ψ_n-type, resp.) recursive function. Similarly, if we set

$$\tilde{\varphi}_n(\xi) := \begin{cases} -2\xi, & \text{if } n \geq 3 \text{ is odd} \\ 1 - 2\xi, & \text{if } n \text{ is even,} \end{cases}$$

$$\left(\tilde{\psi}_n(\xi) := \begin{cases} \frac{a}{\xi}, & \text{if } n \geq 3 \text{ is odd} \\ 1 - \frac{a}{\xi}, & \text{if } n \text{ is even,} \end{cases} \text{resp.}\right)$$

then $g(x) := (a - bx)^{1/2}$ ($g(x) := x^a e^{-b(x+c)}$, resp.) is $\tilde{\varphi}_n$-type ($\tilde{\psi}_n$-type, resp.) recursive function. We note that, if g is fixed point of operator $E^{(1)}$ (i.e., if $g(x) = -\frac{1}{ln(Cx)}$, $C > 0$), then g is n_0-static for every $n_0 \in \mathbf{N}$. On the other hand, we do not know if there exists a function g which is φ-type recursive for any $n_0 \in \mathbf{N}$ provided $\varphi \neq id$, where $id(\xi) := \xi$ is identity function.

Definition 5.2. We say that function a set of functions A is $E^{(n)}$-stable for given $n \in \mathbf{N}$ if it holds that $E^{(n)}(A) \subset A$.

It is easy to see that the set of all rational (irrational, resp.) functions is $E^{(1)}$-stable and therefore $E^{(n)}$-stable for every $n \in \mathbf{N}$. We conjecture that there is no nontrivial set of functions which is $E^{(n)}$-stable for some $n \in \mathbf{N}$, $n > 1$, and which is not $E^{(1)}$-stable.

We note that homogeneity properties of the elasticity coefficients are inherited from homogeneity properties of the function itself.

Theorem 5.1. $x \mapsto g(x)$ is k-homogeneous function for some $k \in \mathbf{R}$ iff $x \mapsto E_{g,x}$ is 0-homogeneous function.

We have not been able to provide direct economic interpretation of higher order elasticity coefficient of general function g. Instead, we introduce some particular classes of functions for which such interpretation is possible. For instance, this is the case for functions g which are φ_n-type recursive with weak dependence of φ_n with respect to n (cf. Example 5.1).

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